國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：微積分【海下海物所碩士班選考】

1. Find the derivative of the following function, (10 %)

$$f(x)=\sqrt[3]{9-x^2}$$

2. Evaluate the integral $\int_2^3 \frac{x}{(1-x)^3}dx$ (10 %)

3. Calculate the square root of 2 by expanding the Taylor series to its 7^{th} derivative, and compute the percentage of error from the result of calculator. (10 %)

4. （10%）D是由圓$x^2+y^2=R^2$所圍成的區域，請計算以下的積分

$$\iint_D e^{-(x^2+y^2)}dxdy$$

5. (a)以 Taylor Series 將$\sqrt[3]{8(1+x)}$對x展開至$O(x^2)$階（5%）
(b)利用以上結果求$\sqrt[3]{9}$的近似值並探討其誤差（10%）

6. 求極限 $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$（5%）。

7. Please evaluate $\int_0^{\pi/2}\frac{\sin x}{2\cos^2 x-3\cos x-2}dx$ (10%)

8. Please find the domain of x such that the following series converges. (10%)
$S_n=1+\tan(x^2)+\tan^2(x^2)+\tan^3(x^2)+\ldots+\tan^n(x^2)$, $n\to\infty$

9. Please evaluate $\int_0^{4\pi}\left|\sin\theta\cos\theta-\frac{1}{2}\right|d\theta$ (10%)

10. Please evaluate $\int\frac{x}{(x+2)(x+3)(x+4)}dx$ (10%)