

1. (20%) 【Derivative】 (5% each)

Find first derivative for each of the following given functions with respect to x , θ or λ :

(a) $f(x) = x^2 e^{-x}$; (b) $x \ln y - y \ln x = 8$;

(c) $f(\theta) = \frac{\theta}{1 - \sin \theta}$; (d) $f(\lambda) = \ln \frac{e^{-\lambda} \lambda^y}{y!}$.

2. (10%) 【Derivative】 (5% each)

Given (a) $x^2 + y^2 = 3$, find $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$;

(b) $z = ye^{2x} + x \ln y^2$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x \partial y}$.

3. (20%) 【Limits】 (5% each)

(a) $\lim_{x \rightarrow 1} \frac{3x - 3}{x^2 - 1}$; (b) $\lim_{x \rightarrow 0} \frac{\sqrt{x} - 1}{x - 1}$;

(c) $\lim_{x \rightarrow 0} \frac{\sqrt{x}}{\sin 3\sqrt{x}}$; (d) $\lim_{x \rightarrow 2^+} \frac{\ln(2x - 3)}{x^2 - 4}$.

4. (30%) 【Integration】 (10% each)

(a) $\int x^2 e^x dx$; (b) $\int \frac{\cos 2x}{\sin^3 2x} dx$; (c) $\frac{d}{dx} \int_{2x}^{x^2} u(1 + u^2)^3 du$.

5. (10%) 【Application】

Let x and y satisfy $x^2 - \sqrt{xy} + y^2 = 6$ and assume that y is a function of x in the neighborhood of $(2, 2)$. Determine the value of $y'(2)$

6. (10%) 【Application】

Water discharges into a large conical tank with its top open. The radius of the top is 5 m and vertical height of the tank is 10 m. If water is running at the constant rate of 2 m^3 per minute, how fast is the water level rising when the water is 6 m deep from its bottom tip?

【Hint: Volume of a conical shape $= \pi r^2 h / 3$, where r is the radius and h the vertical height from bottom where $r = 0$.】