國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：線性代數【海工系碩士班丙組選考】

題號：4175

(1) (20%) Given a matrix: $\begin{bmatrix} 7 & -2 & 1 & -2 \\ 0 & 2 & 6 & 3 \\ 7 & 2 & 13 & 4 \\ 7 & 0 & 7 & 1 \end{bmatrix}$

(a) (10%) Find the reduce form of the matrix and its rank.

(b) (5%) Find a basis for the row space of the matrix, and the dimension of this space.

(c) (5%) Find a basis of the column space of the matrix, and the dimension of this space.

(2) (15%) Find the general solution of the following homogeneous linear system and write it as a sum of column vectors.

$$\begin{cases} -5x_1 + x_2 - 3x_3 + 4x_5 = 0 \\ x_2 - 5x_3 - x_4 + 7x_5 = 0 \end{cases}$$

(3) (15%) Find the general solution of the following non-homogeneous linear system or show that the system has no solution.

$$\begin{cases} 4x_1 - x_2 + 4x_3 = 1 \\ x_1 + x_2 - 5x_3 = 0 \\ -2x_1 + x_2 + 7x_3 = 4 \end{cases}$$

(4) (15%) Produce a matrix that diagonalizes the given matrix: $\begin{bmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{bmatrix}$.

(5) (15%) Use the principal axis theorem to transform the following quadratic form to the conic.

$$4x_1^2 - 4x_2^2 + 6x_1x_2 = 8$$

(6) (20%) The statistics for rainfall in City A are as following:

A Given Day	Following Day Wet
Wet	208 out of 320
Dry	115 out of 500

Use these figures to construct a Markov chain for predicting weather in City A, and answer the following questions.

(a) (10%) If today is a dry Thursday, what is the probability that Saturday (two days later) will also be dry?

(b) (10%) Determine what are the probabilities that a day will be dry and wet in a very long term future?