國立高雄大學 103 學年度研究所碩士班招生考試試題

科目：數理統計　　系所：統計學研究所　　是否使用計算機：否
考試時間：100 分鐘　　本科原始成績：100 分

1. Two players, A and B, alternately and independently flip a coin and the first player to obtain a head wins. Assume player A flips first.
   (a). If the coin is fair, what is the probability that A wins? (5%)
   (b). Suppose that $P(\text{head}) = p$, not necessarily 1/2. What is the probability that A wins? (5%)
   (c). Show that for all p, $0 < p < 1$, $P(\text{A wins}) > 1/2$. (5%)
2. Find $P(|Y-\mu| \le 2\sigma)$ for the exponential random variable. Compare with the corresponding probabilistic statements by Chebyshev's theorem and the empirical rule. (10%)
3. A merchant stocks a certain perishable item. She knows that on any given day she will have a demand for either two, three, or four of these items with probabilities 0.1, 0.4, and 0.5, respectively. She buys the items for $1.00 each and sells them for $1.20 each. If any are left at the end of the day, they represent a total loss. How many items should the merchant stock in order to maximize her expected daily profit? (5%)
4. A member of the Pareto family of distributions (often used in economics to model income distributions) has a distribution function given by

$$F(y) = \begin{cases} 0, & y < \beta \\ 1 - \left(\dfrac{\beta}{y}\right)^{\alpha}, & y \ge \beta, \end{cases}$$

   where $\alpha, \beta > 0$.
   (a). Find the density function. (5%)
   (b). For fixed values of β and α, find a transformation *G(U)* so that *G(U)* has a distribution function of F when U has a uniform distribution on the interval (0,1). (5%)
   (c). Given that a random sample of size 5 from a uniform distribution on the interval (0,1) yielded the values 0.0058, 0.2048, 0.7692, 0.2475 and 0.6078, use the transformation derived in (b) to give values associated with a random variable with a Pareto distribution with $\alpha = 2, \beta = 3$. (5%)
5. One observation is taken on a discrete random variable X with pmf $f(x|\theta)$, where $\theta \in \{1,2,3\}$. Find the MLE of θ. (10%)

x	$f(x\|1)$	$f(x\|2)$	$f(x\|3)$
0	1/3	1/4	0
1	1/3	1/4	0
2	0	1/4	1/4

3	1/6	1/4	1/2
4	1/6	0	1/4

6. Let $Y_1, Y_2, \cdots, Y_n$ be independent, uniformly distributed random variables on the interval $[0, \theta]$.

   (a). Find the density function of $Y_{(k)}$, the kth-order statistic, where k is an integer between 1 and n. (5%)

   (b). Use the result from (a) to find $E(Y_{(k)})$. (5%)

   (c). Find $Var(Y_{(k)})$. (5%)

   (d). Use the result from (c) to find $E(Y_{(k)} - Y_{(k-1)})$, the mean difference between two successive order statistics. Interpret the result. (5%)

7. Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample from the density function given by

$$f(y \mid \theta) = \begin{cases} \left(\dfrac{1}{\theta}\right) r y^{r-1} e^{-y^r/\theta}, & \theta > 0, y > 0 \\ 0, & \text{elsewhere}, \end{cases}$$

   where r is a known positive constant.

   (a). Find a sufficient statistic for θ. (5%)

   (b). Find the maximum-likelihood estimator of θ. (5%)

   (c). Is the estimator in part (b) an MVUE for θ? (5%)

8. Suppose that an engineer wishes to compare the number of complaints per week filed by union stewards for two different shifts at a manufacturing plant. One hundred independent observations on the number of complaints gave means $\bar{x} = 20$ for shift 1 and $\bar{y} = 22$ for shift 2. Assume that the number of complaints per week on the ith shift has a Poisson distribution with mean θ_i, for $i = 1, 2$. Use the likelihood ratio method to test $H_0 : \theta_1 = \theta_2$ versus $H_a : \theta_1 \neq \theta_2$ with $\alpha \approx 0.01$. (10%)