國立交通大學 103 學年度碩士班考試入學試題

科目：統計學(5132)　　考試日期:103年2月15日 第 2 節

系所班別：資訊管理與財務金融學系　　組別：財金碩乙A組　　第 1 頁,共 3 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (8%) Suppose that $X_1,\ldots,X_n$ are random variables for which Var (X_i) has the same value σ^2 for $i=1,\ldots,n$ and $\rho(X_i,X_j)$ has the same value ρ for every pair of values i and j such that $i \neq j$. Prove that $\rho \geq -\frac{1}{n-1}$.

2. (10%) Suppose that X and Y have a continuous joint distribution for which the joint p.d.f. is as follows:

$$f(x,y)=\begin{cases} x+y & \text{for } 0\leq x\leq 1 \text{ and } 0\leq y\leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Please find E(Y|X) and Var(Y|X).

3. (14%) It is very common to use normal distribution to model logarithms of random variables in financial modeling. Let $Y=\log(X)$ and the random variable Y has a normal distribution with parameters μ and σ^2.
   (a) (6%) Determine the probability density function (p.d.f.) of X.
   (b) (8%) Find $E(X)$ and $Var(X)$.
   (Note: The p.d.f. of a normal random variable Y with parameters μ and σ^2 is

$$g(y)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\} \text{ for } -\infty<y<\infty)$$

4. (18%)
   (a) (5%) In testing whether $\beta_1=0$ in a simple regression model $Y_i=\beta_0+\beta_1X_i+\varepsilon_i$, please explain the relation between t-test statistic and F-statistic.

   (b) The following is the summary for a multiple regression model with two independent variables X_1 and X_2, and 13 observations.

Source	Sum of Squares
Regression	100
Error	100
Total	200

   If SSR(X_1)=70 and SSR(X_2) =40,
   (i) (4%) Determine whether there is a significant relationship between dependent variable Y and each independent variable at the 0.05 level of significance.
   (ii) (4%) Compute the coefficients of partial determination, $r^2_{Y1.2}$ and $r^2_{Y2.1}$, and interpret their meanings.

   (c) (5%) Please explain (i) the potential problem of collinearity of the multiple regression analysis, and (ii) how to measure collinearity.

5. (10%) Let X follow a distribution with probability density function (p.d.f.)

$$f(x)=\begin{cases} 2/x^2, & x>1 \\ 0, & \text{elsewhere} \end{cases}.$$

   (a) (4%) Find the probability $P(1<X<3)$.
   (b) (6%)Find the mean of X. Can you use Chebyshev's inequality to approximate the probability in (a)?

6. (10%) On the basis of a random sample the null hypothesis $H_0 : \mu = \mu_0$ is tested against $H_1 : \mu > \mu_0$ and the null hypothesis is not rejected at the 5% significance level.
   (a) (5%) Does this necessary imply that μ_0 is contained in the 95% confidence interval for μ?
   (b) (5%) Does this necessary imply that μ_0 is contained in the 90% confidence interval for μ if the observed sample mean is large than μ_0?
   You must give reasons for your answers.

7. (20%) Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu,100)$.
   (a) (7%) To test $H_0 : \mu = 230$ against $H_1 : \mu > 230$, find the critical region specified by the likelihood ratio test.
   (b) (7%) Is this test uniformly most powerful? Justify your answer.
   (c) (3%) If a random sample of $n = 16$ yielded $\bar{x} = 232.6$, is H_0 rejected at a significance level of $\alpha = 0.1$?
   (d) (3%) Find the p-value of this test.

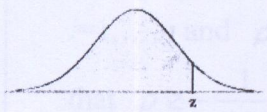
8. (10%) Let $Z_1, Z_2, \cdots, Z_6$ be independent and let each have a standard normal distribution.
   (a) (2%) Specify the distributions of $Y_1 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2$ and $Y_2 = Z_5^2 + Z_6^2$.
   (b) (2%) Specify the distribution of $\dfrac{Y_1}{Y_2}$.
   (c) (2%) Specify the distribution of $Y_1 + Y_2$.
   (d) (2%) Specify the distribution of $\dfrac{Z_5}{\sqrt{\dfrac{Y_1}{4}}}$.
   (e) (2%) Specify the distribution of $\dfrac{Z_5 - Z_6}{\sqrt{\dfrac{Y_1}{8}}}$.

國立交通大學 103 學年度碩士班考試入學試題

科目：統計學(5132)　　考試日期:103年2月15日 第 2 節

系所班別：資訊管理與財務金融學系　　組別：財金碩乙A組　　第 3 頁,共 3 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！



Tables of the Normal Distribution

Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Critical Values of the F-Distribution: $\alpha = 0.05$

Denom. d.f.	Numerator Degrees of Freedom 1	2	3	4	5	6	7
1	161.448	199.500	215.707	224.583	230.162	233.986	236.768
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707