

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：工程數學【海工系碩士班甲組】

題號：4176
共 1 頁 第 1 頁

1. 【Ordinary differential equations】 10%

Test the exactness of the differential equation $2x \tan y \, dx + \sec^2 y \, dy = 0$. If it is exact, solve it. If not, find an integrating factor and solve it.

2. 【Laplace transform】 15%

Solve the IVP using Laplace transform: $y'' + 3y' + 2y = \begin{cases} 4t, & \text{if } 0 < t < 1 \\ 8, & \text{if } t > 1 \end{cases}$, $y(0) = y'(0) = 0$

3. 【Linear algebra】 10%

Given the quadratic form $4x_1^2 + 6x_1x_2 - 4x_2^2 = 10$, transform it to the corresponding conic section (using principal axes).

4. 【Vector calculus】 15%

Find the flux of $\mathbf{F} = [e^y, -e^z, e^x]$ over the surface $S: x^2 + y^2 = 25, x \geq 0, y \geq 0, 0 \leq z \leq 2$ using the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$.

5. 【Method of separation of variables】 15%

Find ALL possible solutions for PDE $\frac{\partial^2 u(x, y)}{\partial x^2} = \frac{\partial u(x, y)}{\partial y}$ by method of separation of variables.

[Note: Please include the constant coefficients in the solutions, since boundary conditions are not specified.]

6. 【Fourier analysis】 15%

Find a Fourier series solution for a periodic square wave given by the function:

$$f(x) = 0, \quad -2 < x < -1$$

$$f(x) = k, \quad -1 < x < 1; \quad p = 2L = 4, L = 2$$

$$f(x) = 0, \quad 1 < x < 2.$$

in (a) series approximation (5%), and (b) expand the series to the term including $n = 5$ (10%).

7. 【Residue integration】 10%

Evaluate the integral $\oint_C \frac{e^{-z^2}}{\sin 4z} \, dz$.

8. 【Modeling】 10%

The un-damped forced spring oscillation can be modeled as the following 2nd order differential equation. Solve the problem by finding the spring displacement $y(t)$.

$$y'' + \omega_0^2 y = \cos \omega t, \quad y(0) = y_0, \quad y'(0) = v_0, \quad \text{and } \omega^2 \neq \omega_0^2$$