

國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：工程數學（僅含微分方程及線性代數）

適用系所：電機工程學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (10分) Find the general solution of $y' + y = x$.
2. (10分) $y''' - 9y'' + 27y' - 27y = 54 \sin(3x)$, $y(0) = 3.5$, $y'(0) = 13.5$, $y''(0) = 38.5$. Find $y = ?$
3. (10分) Find the general solution of $y'' - 5y' + 6y = -3 \sin(2x)$.
4. (10分) Find the current in the RLC circuit in Fig. 1, where $L = 0.1$ H, $R = 20 \Omega$, $C = 2 \cdot 10^{-4}$ F, and $E(t) = 110 \sin(415t)$ volts. Assume zero initial current and capacity charge.

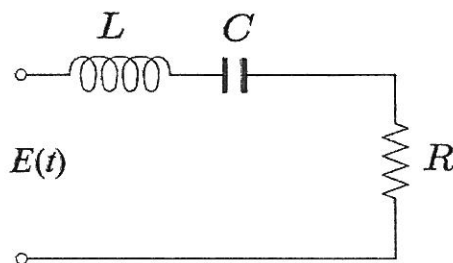


Fig. 1

5. (10分) Find $x(t)$ and $y(t)$ of the following differential equations:

$$\begin{cases} 2 \frac{dx}{dt} - x + \frac{dy}{dt} + 4y = 1 \\ \frac{dx}{dt} - \frac{dy}{dt} = t - 1 \end{cases}$$

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6. (10 分) Let S be the subspace of R^4 spanned by $\langle 2, 0, 2, 0 \rangle$ and $\langle 0, 0, 4, 2 \rangle$. Find the vector in S closest to $\langle -2, 2, -6, 6 \rangle$.
7. (10 分) Suppose A is an orthogonal matrix. Is it true that $|A| = \pm 1$? Justify your answer.
8. Determine whether each of the following statements is true or false. Justify your answer.
- (a) The set $W = \{(x, y, z) \in R^3 : x + 2y - z = 2\}$ is a subspace of R^3 . (5 分)
 - (b) Let $T: R^2 \rightarrow R^2$ be a function defined by $T(x, y) = (x + 2, y)$. The function T is a linear transformation. (5 分)
 - (c) Let V be the space spanned by $u_1 = 2(3 - x)^2$, $u_2 = 2x^2 - 12x$, and $u_3 = 12$. $W = \{u_1, u_2, u_3\}$ is a basis for V . (5 分)
 - (d) Suppose λ is an eigenvalue of the matrix A with eigenvector V . Then, λ^k is an eigenvalue of A^k with eigenvector V where k is a positive integer. (5 分)
9. (10 分) Let $T: R^3 \rightarrow R^3$ be the linear operator given by

$$T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} -2x + y - z \\ x - 2y - z \\ -x - y - 2z \end{bmatrix}$$

Find a basis for R^3 relative to which the matrix for T is diagonal.