國立臺灣師範大學 103 學年度碩士班招生考試試題

科目：**機率與統計**   適用系所：**數學系**

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

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1. (20 points)Let $\{X_i, i = 1, 2, \cdots\}$ be a sequence of i.i.d. exponential random variables with mean $\frac{1}{\lambda}$. Let N be a geometric random variable with parameter p and it is independent of the sequence $\{X_i\}$.

    (a) What is the distribution of $X_1 + X_2 + X_3$? What is its mean?

    (b) Find the mean and variance of $\sum_{i=1}^{N} X_i$.

2. (20 points)Two red balls with labels randomly selected from $U(2, 3)$ distribution. Two blue balls with labels randomly selected from $U(1, 4)$ distribution. Mix these four balls in a box.

    (a) If you pick up two balls from the above four randomly, what is the probability that these two balls have the same color?

    (b) Find the probability that the ball (among all balls) with the largest number in the label is blue.

3. (20 points)Let $0 < p_1, p_2, p_3 < 1$ and $p_1 + p_2 + p_3 = 1$. Let X_1, X_2, X_3 be independent with $n(\mu_i, \sigma_i^2), i = 1, 2, 3$ distributions respectively.

    (a) Let $Y = \sum_{i=1}^{3} p_i X_i$. Find the distribution of Y.

    (b) Let Z has pdf $f(z) = \sum_{i=1}^{3} p_i f_i(z)$, where $f_i(x)$ is the pdf of $X_i, i = 1, 2, 3$. Find the mean and variance of Z.

4. (20 points)Let X be a random variable has the pdf $f_X(x; \theta) = 1/(2\theta)$, for $-\theta < x < \theta$, zero elsewhere.

    (a) Show that $Y = |X|$ is a complete sufficient statistic for $\theta > 0$.

    (b) Show that $Y = |X|$ and $Z = sgn(X)$ are independent where $sgn(t) = 1, 0,$ or -1 depending on whether $t > 0, t = 0,$ or $t < 0$.

5. (20 points)In order to compare the relationship between weight (pounds) and height (inches) for males and females, some researcher collected data of m males and n females and obtained the following regression lines, respectively. (y=weight and x=height)

    Male: $\hat{y} = b_0^M + b_1^M x$ for $E(Y|X) = \beta_0^M + \beta_1^M X$

    Female: $\hat{y} = b_0^F + b_1^F x$ for $E(Y|X) = \beta_0^F + \beta_1^F X$

    Suppose all the necessary assumptions are satisfied, the t-statistics for testing $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$ obtained for b_1^M and b_1^F are respectively t_M and t_F. To conduct the hypothesis test of $H_0 : \beta_1^M = \beta_1^F$ versus $H_1 : \beta_1^M \neq \beta_1^F$, one can simply use the above results.

    (a) Define the test statistic you would use and its relevant distribution under the null hypothesis.

    (b) For a given significance level α, how would you draw the decision of whether such a relationship differs between males and females.