國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：微積分【公事所碩士班甲組選考】

題號：4130

請依題號順序作答，違者扣分。

1. Find the derivatives for the following functions. 20%

   i. $f(x)=\dfrac{1}{1+e^{-x}}$ ii. $y=e^{-\pi}+\pi^{-e}$ iii. $f(x)=\dfrac{x}{\cos^2 x}$

   iv. Find dy/dx where $\sin(3y)+\cos(5x)=xy$

2. Sketch the function. $f(x)=x+\sin x,(0\le x\le 2\pi)$. 20% Indicate

   i. the critical points,
   ii. the reflection point(s),
   iii. the global maxima and minima,
   iv. the increasing and decreasing and concavity of the function

3. Evaluate the following. 30 %

   i. $\int xe^{-x^2}dx$ ii. $\lim_{x\to\infty} xe^{-x}$ iii. $\int_0^6\int_{x/3}^2 x\sqrt{y^3+1}dydx.$

   iv. $\int_0^1 xf''(x)dx$ where f is twice differentiable with $f(0)=6$, $f(1)=5$, and $f'(1)=2$.

   v. $\int_e^\infty x^p \ln x dx$. indicate the value of p so that the integral converges.

   vi. Determine if the sequences converge a. $\sum_{n=1}^{\infty}\dfrac{n-1}{n^3+3}$ b. $\sum_{n=1}^{\infty}\sin(\dfrac{1}{n})$

4. By looking at their Taylor series, decide which of the following functions is the largest and which is the smallest, for x near 0. 15%

   i $1+\sin x$ ii. e^x iii. $\dfrac{1}{\sqrt{1-2x}}$

5. While taking a walk along the road where you live, you accidentally drop your i-phone, but you don't know where. The probability density $p(x)$ for having dropped the i-phone x kilometers from home (along the road) is $p(x)=2e^{-2x}$ for $x\ge 0$. 15%

   i. What is the probability that you dropped it within 1 kilometer of home?
   ii. At what distance y from home is the probability that you dropped it within y km of home equals to 0.95