

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：微積分丁【企管系甲班碩士班丁組選考】

題號：4112

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請依題號順序作答，違者扣分。

1. Find the derivatives for the following functions. 20%

i.  $f(x) = \frac{1}{1+e^{-x}}$     ii.  $y = e^{-\pi} + \pi^{-e}$     iii.  $f(x) = \frac{x}{\sin^2 x}$

iv. Find  $dy/dx$  where  $\sin(3y) + \cos(5x) = xy$

2. Sketch the function  $f(x) = x + \sin x, (0 \leq x \leq 2\pi)$  20%

Indicate

- i. the critical points,
- ii. the reflection point(s),
- iii. the global maxima and minima,
- iv. the increasing and decreasing and concavity of the function

3. Evaluate the following. 25 %

i.  $\int xe^{-x^2} dx$     ii.  $\int_2^3 \ln x dx$     iii.  $\lim_{x \rightarrow \infty} xe^{-x}$     iv.  $\int_0^6 \int_{x/3}^2 x\sqrt{y^3+1} dy dx.$

v.  $\int_e^{\infty} x^p \ln x dx$ , indicate the value of  $p$  so that the integral converges.

4. Determine if the following sequences converge. 10%

i.  $\sum_{n=1}^{\infty} \frac{n-5}{n^3+8}$     ii.  $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

5. By looking at their Taylor series, decide which of the following functions is the largest and which is the smallest, for  $x$  near 0. 15%

i.  $e^x$     ii.  $1 + \sin x$     iii.  $\frac{1}{\sqrt{1-2x}}$

6. 某宅配公司的運費以包裹的長寬高長度總和來計價。如果某包裹的長寬高的長度總和限制在 60 公分的話，請問此包裹的最大容量為何？（請列出計算過程）10%