1. **[20] Matched Filter:** Prove that if a signal $s(t)$ is corrupted by AWGN, the filter with an impulse response matched to $s(t)$ maximizes the output signal-to-noise ratio. The maximum SNR obtained with the matched filter is $\mathrm{SNR}_0 = \frac{2}{N_0}\int_0^T s^2(t)\,dt = \frac{2\varepsilon}{N_0}$.

2. **[20] Hilbert Transform:** The Hilbert transform is given by $\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}d\tau$. Prove the following properties:
   A. [5] If $x(t) = x(-t)$, then $\hat{x}(t) = -\hat{x}(-t)$.
   B. [5] If $x(t) = -x(-t)$, then $\hat{x}(t) = \hat{x}(-t)$.
   C. [5] $\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = 0$.
   D. [5] If $x(t) = \sin\omega_0 t$, then $\hat{x}(t) = -\cos\omega_0 t$.

3. **[20] *M*-ary PAM Modulation:** The *M*-ary PAM signals can be represented geometrically as *M* one-dimensional signal points with value $s_m = \sqrt{\frac{1}{2}\varepsilon_g} A_m$, $m = 1,2,\ldots,M$, where ε_g is the energy of the basic signal pulse $g(t)$. The amplitude values may be expressed as $A_m = (2m-1-M)d$, $m = 1,2,\ldots,M$. On the assumption of the each signal has equal probability,
   A. [5] Find the average energy.
   B. [10] Calculate the average probability of a <u>symbol error</u> for *M*-ary PAM.
   C. [5] Please use the result in Part A to show the probability of a symbol error for rectangular *M*-ary QAM. ($M = 2^k$, k is even)

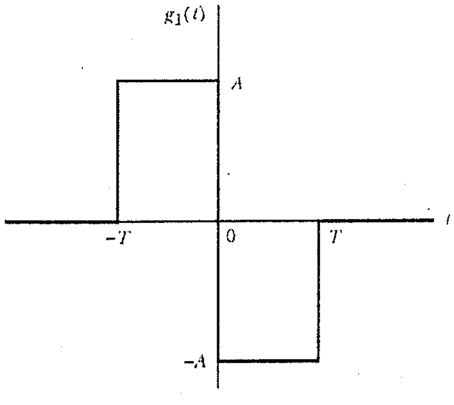
4. **[20] Band-Pass Systems**: Consider a band-pass system. The time domain received signal is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$, where $h(t) = \mathrm{Re}\left[\tilde{h}(t)\exp(j2\pi f_c t)\right]$ is the impulse response of a bandpass filter and $\tilde{h}(t)$ is the complex impulse response of the bandpass filter.
   A. [8] Please show that $H(f) = \frac{1}{2}\left[\tilde{H}(f-f_c) + \tilde{H}^*(-f-f_c)\right]$, where $H(f)$ and $\tilde{H}(f)$ are Fourier transform of $h(t)$ and $\tilde{h}(t)$, respectively.
   B. [12] Please show that $\tilde{y}(t) = \frac{1}{2}\tilde{h}(t) * \tilde{x}(t)$, where $\tilde{x}(t)$ and $\tilde{y}(t)$ are the complex envelope of the band-pass input and output, respectively.

5. **[20] Fourier Transform:** (Hint: You may use the attached properties of the Fourier transform.)

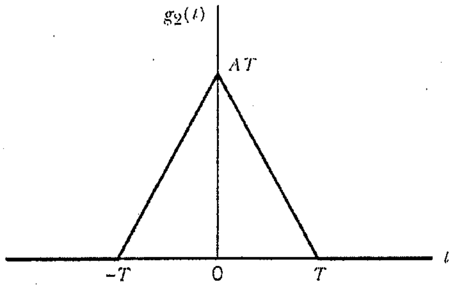
A. [5] Find the Fourier transform of the rectangular pulse: $g(t)=\begin{cases}1, & -\frac{1}{2}\le t\le\frac{1}{2}, \\ 0, & |t|\ge\frac{1}{2}.\end{cases}$

B. [5] Find the Fourier transform of the doublet pulse $g_1(t)$ shown in Figure (a).

C. [10] Find the Fourier transform of the triangular pulse $g_2(t)$ shown in Figure (b).



(a)



(b)

Properties of the Fourier Transform

Property	Mathematical Description
Linearity	$ag_1(t)+bg_2(t)\rightleftharpoons aG_1(f)+bG_2(f)$.
Time scaling	$g(at)\rightleftharpoons\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$, where a is a constant.
Duality	If $g(t)\rightleftharpoons G(f)$, then $G(t)\rightleftharpoons g(-f)$.
Time shifting	$g(t-t_0)\rightleftharpoons G(f)\exp(-j2\pi ft_0)$.
Frequency shifting	$\exp(j2\pi f_c t)g(t)\rightleftharpoons G(f-f_c)$.
Area under $g(t)$	$\int_{-\infty}^{\infty}g(t)\,dt=G(0)$.
Differentiation in the time domain	$\frac{d}{dt}g(t)\rightleftharpoons j2\pi fG(f)$.
Integration in the time domain	$\int_{-\infty}g(\tau)\,d\tau\rightleftharpoons\frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$.
Conjugate functions	If $g(t)\rightleftharpoons G(f)$, then $g^*(t)\rightleftharpoons G^*(-f)$.
Multiplication in the time domain	$g_1(t)g_2(t)\rightleftharpoons\int_{-\infty}^{\infty}G_1(\lambda)G(f-\lambda)\,d\lambda$.
Convolution in the time domain	$\int_{-\infty}g_1(\tau)g_2(t-\tau)\,d\tau\rightleftharpoons G_1(f)G_2(f)$.
Rayleigh's energy theorem	$\int_{-\infty}^{\infty}\lvert g(t)\rvert^2dt=\int_{-\infty}^{\infty}\lvert G(f)\rvert^2df$.