國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：機率【通訊所碩士班甲組】

題號：4089

1. (15 %) For each part, if the statement is true, please write a circle ("o"). If the statement is wrong, then mark it as ("x"). You do NOT need to provide any justification.
   (a) (5 %) (   ). Let X, Y be independent random variables, both uniformly distributed on (-1/2,1/2). Then $X+Y$ is uniformly distributed on (-1,1).
   (b) (5 %) (   ). Assume X, Y be independent random variables, both normally distributed with parameters (μ,σ^2) being $(2,3^2)$ and $(-2,4^2)$. Then $X+Y$ is normally distributed with parameters $(0,5^2)$.
   (c) (5 %) (   ). Let the joint density function of X, Y be $f(x,y)=\frac{4}{\pi}\exp\left(-(x^2+y^2)\right)$ for $x>0, y>0$, and zero otherwise. Then X and Y are independent.

2. (10 %) Assume a random variable X is uniform on $(0,L)$. Decide the probability of which new variable $\min\left(\frac{X}{L-X},\frac{L-X}{X}\right)$ is less than 1/3. (i.e., calculate $P\left(\min\left(\frac{X}{L-X},\frac{L-X}{X}\right)<\frac{1}{3}\right)$)

3. (15%) Consider two random variables X and Y with the joint distribution $f(x,y)=ce^{-(\pi x^2+4\pi y^2)}$, $-\infty<x,y<\infty$. Please decide
   (a) (5 %) c;
   (b) (5 %) $P\left(Y>0 \mid X>\frac{1}{\pi}\right)$;
   (c) (5 %) $E(XY \mid Y=\pi)$.

4. (10%) A random variable X is uniform on (-2,3). If $Y=-X^2+4$, find the distribution of Y.

5. (15%) Given any two real-valued random variables X_1 and X_2 with finite second moment. Here, $\mathrm{E}\{\cdot\}$ takes the expectation with respect to X_1 and X_2. If the statement is true, please write a circle ("o"). If the statement is wrong, then mark it as ("x"). You do NOT need to provide any justification.
   (a) (5 %) (   ). $(\mathrm{E}\{X_1X_2\})^2 \le \mathrm{E}\{X_1^2\}\mathrm{E}\{X_2^2\}$;
   (b) (5 %) (   ). $\mathrm{E}\{c_1X_1+c_2X_2\} \ne c_1\mathrm{E}\{X_1\}+c_2\mathrm{E}\{X_2\}$, where c_1 and c_2 are constant values;
   (c) (5 %) (   ). $\mathrm{E}\{(X_1+X_2)^2\} \le \mathrm{E}\{X_1^2\}+\mathrm{E}\{X_2^2\}$.

6. (15%) Let Y be a binomial distribution with parameters n and p; i.e., the probability distribution function of Y is given by $P(Y=y)=\binom{n}{y}p^y(1-p)^{n-y}$, $y=0,1,2,\cdots,n$. Please find
   (a) (5 %) the mean of Y,

(b) (5 %) the variance of Y,

(c) (5 %) the probability generating function of Y.

7. (10%) The joint probability density function of the random variable (X_1, X_2) is given by

$$f(x_1, x_2) = \begin{cases} c(x_1 + x_2) & 0 < x_1 < x_2 < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Are X_1 and X_2 stochastically independent? Why?

8. (10%) Let X and Y be independent normal random variables with zero mean and unit variance. Find the value of $\mathrm{E}\{X^2Y + XY^2 + X^2Y^2\}$, in which $\mathrm{E}\{\cdot\}$ takes the expectation with respect to X and Y.