國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：工程數學【光電所碩士班】

題號：4082

1. (35%) Find respective general solutions for following equations

(a) $e^{-ay}dx + \frac{1}{x}dy = 0$ (5%)

(b) $(x^2 + 3y^2)\,dx - 2xy\,dy = 0$ (6%)

(c) $\frac{dy}{dx} = xe^{(x-y)}$ (6 %)

(d) $\frac{dy}{dx} = \frac{y(1+2xy)}{x(xy-1)}$ (6%)

(e) $y'' - 2y' + y = x - 2$ (6%)

(f) Prove: $\nabla \bullet \left(\frac{\mathbf{r}}{r^3}\right) = 0$ (6%)

2. (10%) Evaluate following equation with boundary conditions $u(0, y) = e^{-y}$

$$\frac{\partial u(x,y)}{\partial x} + \frac{\partial u(x,y)}{\partial y} = u(x,y), \qquad x > 0, y > 0.$$

3. (15%) For $z = x + iy$, solve following equations.

(a) $\oint_c \frac{dz}{z^2 - 2z + 2}$, c: $|z - (2 + i2)| = 2$. (7 %)

(b) Let $f(z) = x^2 + iy^2$, evaluate (8 %)

$\int_c f(z)dz$,

where c is a curve $y = \cos x$ from $x = 0$ to $x = \pi/2$.

4. (10%) Find the inverse matrix for

S = TA,

where $T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $A = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$.

5. (15%) If $R(z) = u(x, y) + iv(x, y)$ is an analytic function, and $u(x, y) = \exp(3x)\cos 3y$, (a) find out $v(x, y)$ for $R(z)$ (7%), and (b) calculate R' (8%).

6. (15%) The distribution of surface energy of a thin film is

$\sigma = x^2 + y^2 + 2xz$.

At point (2, 1, 0), find (a) energy gradient (5%), (b) the unit vector in the direction of the energy gradient (5%), (c) the curl of the surface force (5%).