

1. Find  $\lim_{x \rightarrow \frac{\pi}{2}} \ln \left( \frac{\tan x + 1}{x + \sec x} \right)$ . (10%)
2. Evaluate  $\int_0^1 x \tan^{-1} x \, dx$ . (10%)
3. Evaluate  $\int_0^1 \frac{dx}{e^x - e^{-x}}$ . (10%)
4. (a) Show that  $\int_1^3 e^{-xy} \, dy = \frac{1}{x}(e^{-x} - e^{-3x})$ . (5%)
- (b) Evaluate  $\int_0^{\infty} \frac{1}{x}(e^{-x} - e^{-3x}) \, dx$ . (5%)
5. Find an equation of the tangent line to the graph of the equation  $y = \int_1^x \sqrt{4 - u^3} \, du$  at  $x = 1$ . (10%)
6. (a) Find the Maclaurin series for  $\cos x$ . (5%)
- (b) Explain why  $\left| \frac{1 - \cos x}{x} \right| \leq \frac{1}{2}|x|$ ,  $\forall x \neq 0$ . (5%)
7. Find the length of the loop of the graph of  $x = 3t^2$ ,  $y = t^3 - 3t$ . (10%)
8. Determine and classify the stationary points of the implicit function  $z = z(x, y)$  defined by the equation  $x^2 + y^2 + 2z^2 + 8xz + 7 = 0$ . (10%)
9. The temperature at each point of the circular disk  $D = \{(x, y) : x^2 + y^2 \leq 1\}$  is given by  $T = x^2 + y^2 - x$ . Find the hottest and coldest points of the disk. (10%)
10. Let  $R$  be the region bounded by the parabola  $y = x^2 + 1$  and the line  $y = x + 1$ . Evaluate  $\iint_R \frac{\sin x}{x} \, dA$ . (10%)