國立雲林科技大學
97學年度碩士班入學招生考試試題

系所：通訊所、電機系 丙組
科目：線性代數

1. (10%) Let $\mathbf{u}$ be a vector in $\mathbf{R}^2$ whose projection onto the $x-axis$ is u_x as shown in Figure 1. Determine the entries of the vector $\mathbf{u}$.

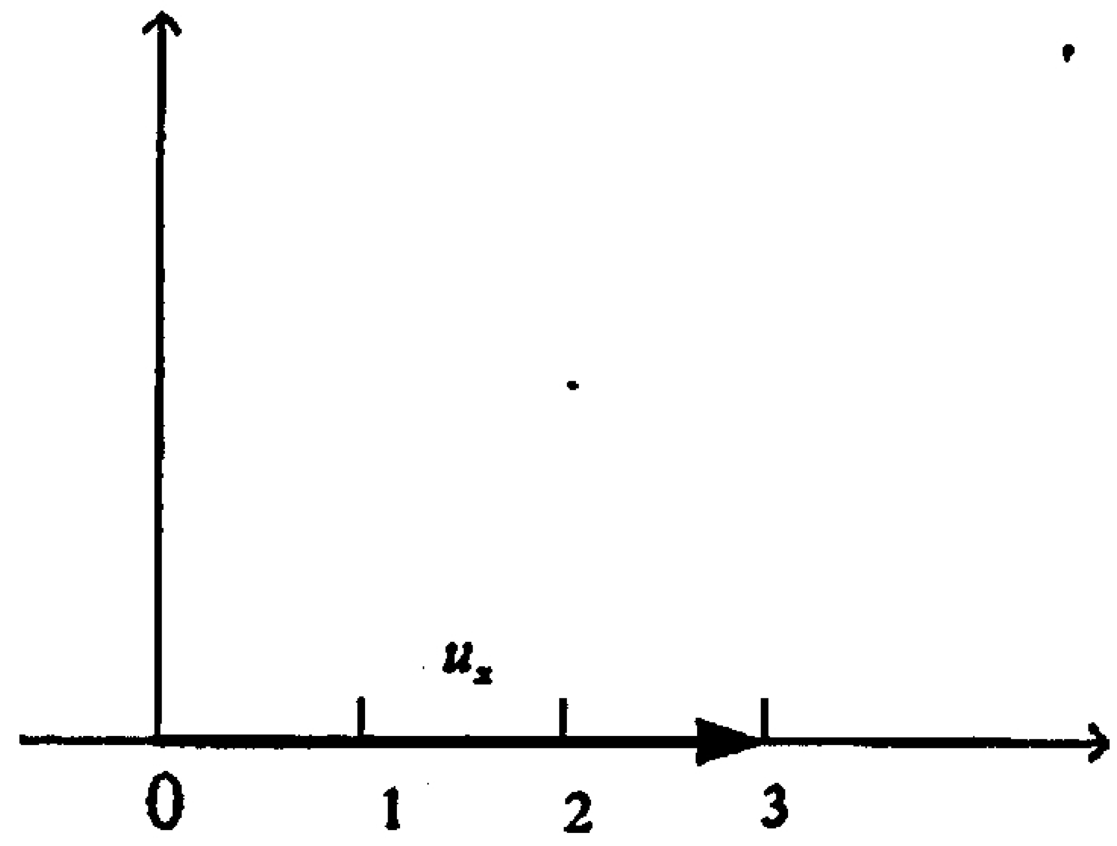

Figure 1.

2. (10%) Let

$$\mathbf{A} = \begin{bmatrix} 6 & 2 & 8 \\ 9 & 5 & 11 \\ 3 & 1 & 6 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 2 & 0 & 0 \\ t & s & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} r & 1 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & p \end{bmatrix}$$

Find scalars r, s, t and p so that $\mathbf{LU} = \mathbf{A}$.

3. (15%) Determine whether each of the following statements is *True* or *False*, and explain.

(a) $\det(A+B) = \det(A) + \det(B)$

(b) $\det(A^{-1}B) = \frac{\det(B)}{\det(A)}$

(c) If $\det(A) = 0$, then A has at least two equal rows.

(d) If A has a column of all zeros, then $\det(A) = 0$.

(e) A is singular if and only if $\det(A) = 0$.

4. (15%) Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Show that $span(S) = \mathbf{R}^3$ and find a basis for $\mathbf{R}^3$ consisting of vectors from S.

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5. (15%) Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation for which we know that

$$L\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)=\begin{bmatrix}1\\2\\3\end{bmatrix},\ L\left(\begin{bmatrix}0\\1\\2\end{bmatrix}\right)=\begin{bmatrix}1\\0\\0\end{bmatrix},L\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\0\\1\end{bmatrix}$$

(a) Find $L\left(\begin{bmatrix}4\\1\\0\end{bmatrix}\right)$ =?

(b) Find $L\left(\begin{bmatrix}0\\0\\0\end{bmatrix}\right)$ =?

6. (10%) Consider vector space $\mathbf{R}^2$.

(a) For what values of m and b will the set of all vectors of the form $\begin{bmatrix}x\\mx+b\end{bmatrix}$ be a *subspace* of $\mathbf{R}^2$?

(b) For what value of r will the set of all vectors of the form $\begin{bmatrix}x\\rx^2\end{bmatrix}$ be a *subspace* of $\mathbf{R}^2$?

7. (10%) Let L: $\mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by

$$L\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{bmatrix}x_2-x_1\\2x_1+x_2\end{bmatrix},$$

and let

$$S = \left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}$$
$$T = \left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix}\right\}$$

be two bases for $\mathbf{R}^2$. Find the matrix representation $[L]_T^S$ of L with respect to T and S.

8. (15%) Let $A=\begin{bmatrix}3&1\\0&2\end{bmatrix}$.

(a) Find a nonsingular matrix P such that $P^{-1}AP$ is diagonal.

(b) Derive a formula for A^k, where k is any positive integer.