

# 國立彰化師範大學 97 學年度碩士班招生考試試題

系所：數學系碩士班

組別：丁組

科目：微積分

☆☆請在答案紙上作答☆☆

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請回答下列各題(配分如各題所示，共 100 分)

1. (16%)

Find the minimum value of  $x^2 + y^2 + z^2$  subject to the constraints  $x - y = 2$  and  $y^2 + 2y - z = -2$ .

2. (16%)

Evaluate the indefinite integral  $\int \sqrt{3x^2 + 4x + 1} dx$ .

3. (10%)

Evaluate the integral  $\int_0^1 \int_{y/3}^1 \sin(x^2) dx dy$ .

4. (24%)

Determine whether the following series converge or diverge. Give reasons for your answers.

$$(a) \sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!} \quad (b) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/2}} \quad (c) \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}.$$

5. (12%)

Show that the function

$$f(x, y) = \begin{cases} \frac{3xy^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous at  $(x, y) = (0, 0)$ .

6. (12%)

Use change of variables in a double integral to evaluate the integral

$$\iint_R (x + y)^2 e^{(y-x)} dy dx$$

where  $R$  is the region inside the square with vertices  $(0, 0)$ ,  $(1, 1)$ ,  $(0, 2)$ , and  $(-1, 1)$ .

7. (10%)

Evaluate each limits.

$$(a) \lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{1-\cos(2x)} \quad (b) \lim_{x \rightarrow 0} \frac{\sin(x) - \tan(x)}{x^3}$$