

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：工程數學【機電系碩士班甲組、乙組、丙組、丁組、戊組】

題號：4092
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I. (35%)

1. Solve the following ODEs. Find the general solution or the particular solution by using the given initial conditions. Show the detail of your work.

- (a) $yy'' = y'^2, y(0) = 1, y'(0) = 2.$ (7%)
- (b) $y' = -\frac{2e^y + 2y}{e^y + 1}.$ (8%)
- (c) $y'' + 2y' + 2y = e^{-x}\cos x.$ (10%)
- (d) $x^3y''' - 3x^2y'' + 6xy' - 6y = x.$ (10%)

II. (35%)

1. (20%)

Find an eigenbasis of matrix A and use similarity transformation to diagonalize it.

$$A = \begin{bmatrix} 1 & 6 \\ 1.5 & 1 \end{bmatrix}$$

2. (15%)

- (a) Given a curve C: $r(t) = [5\cos t, 5\sin t, 0],$ find a tangent vector $r'(t),$ and the tangent of C at P: (4, 3, 0). (10%)
- (b) Sketch the curve and the tangent. (5%)

III. (30%)

1. (10%) This happens if PDE involves derivatives with respect to one variable only, so that the other variables can be treated as parameters.

Solve for $u = u(x, y)$ as list:

- (a) $u_{yy} + 16u = 0$ (5%)
- (b) $u_{xy} = u_x$ (5%)

2. (20%) The faces of the thin square plate in Figure 1 with side $a = 24$ are perfectly insulated. The upper side is kept at 20°C and the other sides are kept at $0^\circ\text{C}.$

- (a) Find the steady-state temperature $u(x, y)$ in the plate. (10%)
- (b) Find the steady-state temperature in the plate with the upper and lower sides perfectly insulated, the left side kept at $0^\circ\text{C},$ and the right side kept at $f(y)^\circ\text{C}.$ (10%)

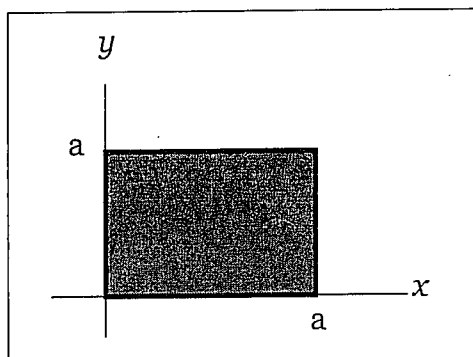


Figure 1: Square Plate