●不可使用電子計算機

1、 Let X equal the forced vital capacity (FVC) in liters for a female college student. (This is the amount of air that a student can force out of her lungs.) Assume that the distribution of X is (approximately) $N(\mu, \sigma^2)$. Suppose it is known that $\mu = 3.4$ liters. A volleyball coach claims that the FVC of volleyball players is greater than 3.4. She plans to test her claim using a random sample of size $n = 9$.

(a) Define the null hypothesis. (5%)
(b) Define the alternative (coach's) hypothesis. (5%)
(c) Define the test statistic. (5%)
(d) Define a critical region for which $\alpha = 0.05$. Draw a figure illustrating your critical region. (5%)
(e) Calculate the value of the test statistic given that the random sample yielded the following forced vital capacities. (5%)

3.4 3.6 3.8 3.3 3.4 3.5 3.7 3.6 3.7

(f) What is your conclusion? (5%)

2、 Let $Y = X_1 + X_2 + \cdots + X_{15}$ be the sum of a random sample of size 15 from the distribution whose p.d.f. is $f(x) = (3/2)x^2$, $-1 < x < 1$. Approximate

$$P(-0.3 \le Y \le 1.5).$$ (10%)

3、 Let X_1 and X_2 be a random sample of size $n = 2$ from a distribution with p.d.f. $f(x) = 6x(1-x)$, $0 < x < 1$. Find the mean and the variance of $Y = X_1 + X_2$. (10%)

4、 Suppose two independent claims are made on two insured homes, where each claim has p.d.f.

$$f(x) = \frac{4}{x^5}, \qquad 1 < x < \infty,$$

in which the unit is 1000 dollars. Find the expected value of the largest claim.
HINT: If X_1 and X_2 are the two independent claims and $Y = \max(X_1, X_2)$, then

$$G(y) = P(Y \le y) = P(X_1 \le y)P(X_2 \le y) = [P(X \le y)]^2.$$

Find $g(y) = G'(y)$ and $E(Y)$. (10%)

5、 The moment-generating function of X is

$$M_X(t) = \left(\frac{1}{4}\right)(e^t + e^{2t} + e^{3t} + e^{4t});$$

the moment-generating function of Y is

$$M_Y(t) = \left(\frac{1}{3}\right)(e^t + e^{2t} + e^{3t});$$

X and Y are independent random variables. Let $W = X + Y$.

(a) Find the moment-generating function of W. (10%)
(b) Give the p.m.f. of W; that is, determine $P(W = w)$, $w = 2, 3, \ldots, 7$, from the moment-generating function of W. (10%)

6. Let X_1, X_2, X_3 be a random sample of size $n = 3$ from an exponential distribution with mean $\theta > 0$. Reject the simple null hypothesis H_0: $\theta = 2$ and accept the composite alternative hypothesis H_1: $\theta < 2$ if the observed sum $\sum_{i=1}^{3} x_i \le 2$. (10%)

(a) What is the power function $K(\theta)$ written as an integral?
(b) Using integration by parts, define the power function as a summation. (10%)

Note: $T \sim t(8)$, $P(T > 1.86) = 0.05$,

$Z \sim N(0,1)$, $P(Z > 0.1) = 0.4602$, $P(Z < 0.5) = 0.6915$