

國立中央大學97學年度碩士班考試入學試題卷

所別：電機工程學系碩士班 電波組 科目：工程數學 共 / 頁 第 / 頁

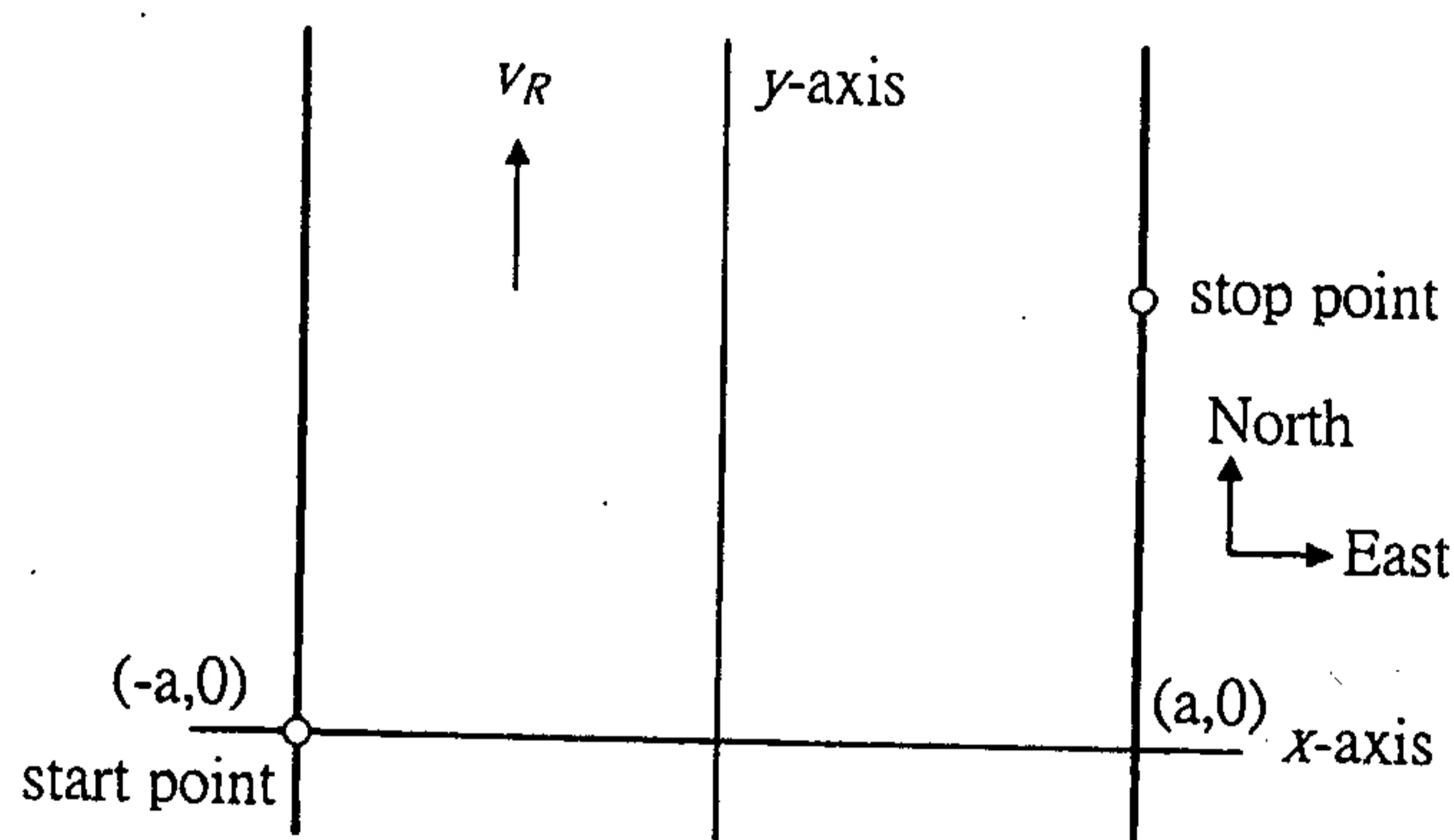
\*請在試卷答案卷(卡)內作答

參考用

1. (20%) The following figure shows a northward-flowing river of width  $w = 2a$ . The lines  $x = \pm a$  represent the banks of the river, and the  $y$ -axis is its center. Suppose that the velocity with which the water flows increases as one approaches the center of the river, and indeed is given in terms of distance  $x$  from the center by

$$v_R = v_0 \left( 1 - \frac{x^2}{a^2} \right)$$

Suppose that a swimmer starts at the points  $(-a, 0)$  on the west bank and swim due east (relative to the water) with a constant speed  $v_s$ . Determine the coordinate of the stop point given the river midstream velocity  $v_0 = 9$  km/hour, the swimmer's velocity  $v_s = 3$  km/hour, and the river width  $w = 2a = 1$  km.



2. (20%) Solve the boundary value problem

$$\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0$$

$$u(0, y) = u(a, y) = u(x, b) = 0.$$

$$u(x, 0) = f(x)$$

3. (20%) Find the eigenvalues and eigenvectors of the matrix  $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ , where  $a, b \in \text{real constants}$ .

4. (20%) Determine  $a$  such that  $u = \sin ax \cosh 2y$  is harmonic, and find the harmonic conjugate of  $u$ .

5. (20%) Evaluate the integral  $\int_0^{2\pi} \frac{d\theta}{k + \cos\theta}$  ( $k > 1$ ).