

國立中央大學97學年度碩士班考試入學試題卷

所別：電機工程學系碩士班 電子組  
 電機工程學系碩士班 固態組  
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科目：工程數學 共 1 頁 第 1 頁

\*請在試卷答案卷(卡)內作答

參考用

1. (15%) Evaluate the following integral. Detailed evaluation procedure is required.

$$\int_0^{\infty} \frac{x \sin(2x)}{x^2 + 3} dx$$

2. (15%) Given the function  $\frac{(3z+1)}{(z-1)}$ , represent it by its Maclaurin series, and give the region of validity for the representation.

3. (15%) Consider the matrix  $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$ . Diagonalize  $A$  by similarity transformation. Transition matrices should be given.

4. (a) If  $\vec{r}$  and  $\vec{n}$  are the position vector and unit normal vector to a closed surface  $S$  of a region  $T$ , which can be non-smooth, evaluate the surface integral of  $\oiint_S \frac{\vec{r}}{r^3} \cdot \vec{n} dS$ , when the coordinate of origin is (i) (5%) outside of  $S$  and

(ii) (5%) inside of  $S$ .

- (b) (5%) Consider a charge  $q$  at the origin. Its corresponding electric field is

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon r^3} \text{ where } \epsilon \text{ is a dielectric constant. Based on the results in (a),}$$

derive Poisson's equation  $\nabla^2\phi = \frac{Q}{\epsilon}$  where  $Q$  is charge density and  $\phi$  is

electrostatic potential such that  $q = \iiint_T Q dV$ , ( $V$ : volume) and  $\vec{E} = \nabla\phi$  respectively.

5. (15%) Solve the following differential equation

$$y''' - 6y'' + 12y' - 8y = \sqrt{x}e^{2x}.$$

6. (15%) Find the Laplace transform of the full-wave rectification of  $\sin \omega t$  (Show the details of your work).

7. (10%) Using the Fourier integral representation, show that

$$\int_0^{\infty} \frac{\cos(\pi\omega/2) \cos x\omega}{1-\omega^2} d\omega = \begin{cases} \frac{\pi}{2} \cos x & \text{when } |x| < \frac{\pi}{2} \\ 0 & \text{when } |x| > \frac{\pi}{2} \end{cases}$$