參考用

(1) Based on the following Schrodinger equation

$i\hbar\dfrac{\partial\psi}{\partial t}=\dfrac{-\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2}+V(x)\psi$, Prove that probability density $\rho=\int_{-\infty}^{\infty}|\psi(x,t)|^2dx$ is a time-independent constant if ψ satisfies the condition of square-integrable. (20%)

(2) Simple harmonic oscillator system can be described by the Schrodinger equation $\dfrac{-\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2}+\dfrac{1}{2}m\omega^2x^2\psi=E\psi$, prove that its Hamiltonian can well be written as $H=\hbar\omega(a^+a+\dfrac{1}{2})$, where $a=\dfrac{1}{\sqrt{2\hbar m\omega}}(ip+m\omega x)$ and $a^+=\dfrac{1}{\sqrt{2\hbar m\omega}}(-ip+m\omega x)$. Note p and x denote, respectively, the momentum and position. (20%)

(3) Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$. Meanwhile, prove that these eigenvectors are orthogonal. (20%)

(4) Density of states $D(\varepsilon)$ means the number of eigenstates in a small interval dε around energy ε. According to such a definition. Describe $D(\varepsilon)$ as a function of ε for different semiconductor heterostructures: quantum wells, wires and dots. (20%)

(5) Write out the Fermi-Dirac distribution function and the Boson-Einstein distribution function. (10%)

(6) Briefly describe what the Bloch theorem is. (10 %)