

1. An electric field may be represented as  $E(\mathbf{x}) = \mathbf{x}/(r^2 + a^2)^{3/2}$ , where  $\mathbf{x}$  denotes a point vector with its magnitude  $r = |\mathbf{x}|$ , and  $a$  is a constant. Compute the divergence of this field, and describe the tangent curves of  $E(\mathbf{x})$ . (15%)
2. A hole of radius  $R$  whose center is at the origin is cut from the  $xy$  plane. The rest of  $xy$  plane has constant surface charge density  $A$  coulombs/cm<sup>2</sup>. Find the  $E$  on the  $z$  axis, and the first two nonvanishing terms of the expansion of  $E_z(0, 0, z)$  for  $z \gg R$ . (15%)
3. Two coupled circuits have self-inductances  $L_1$  and  $L_2$ , that carry currents  $I_1$  and  $I_2$ , respectively. The mutual inductance between the circuits is  $M$ . Find the ratio  $I_1/I_2$ , that makes the total stored magnetic energy a minimum, and show that  $M \leq (L_1 L_2)^{1/2}$ . (20%)
4. If  $(E, H)$  are solutions of source-free Maxwell's equations in a simple medium characterized by  $\epsilon$  and  $\mu$ . Show that the pair of  $E' = \eta H$  and  $H' = -(E/\eta)$  are also solutions of the above case. (15%)
5. Describe a magnetic dipole and define magnetic dipole moment with its SI unit. (15%)
6. Describe the Hall effect, and explain how this effect may be used in measuring the blood flow. (20%)