1. Find general solution of the following ordinary differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-3y=x\sin x \qquad (20\%)$$

2. Find solution of the following partial differential equation

$$-\frac{1}{2\mu_1}\frac{\partial^2\Psi(x,y)}{\partial x^2}-\frac{1}{2\mu_2}\frac{\partial^2\Psi(x,y)}{\partial y^2}=E\Psi(x,y)$$

where μ_1,μ_2 and E are constants. (10%)

3. Find eigenvalues and the corresponding normalized eigenvectors for the following symmetry matrix.

$$\mathbf{A}=\begin{pmatrix}\alpha & \beta & \beta & 0\\ \beta & \alpha & \beta & \beta\\ \beta & \beta & \alpha & \beta\\ 0 & \beta & \beta & \alpha\end{pmatrix}$$

where α and β are constants. (20%)

4. (a) Prove that $\sum_{k=0}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1)$. (10%)

(b) Use the result of (a) to evaluate $\sum_{k=0}^{n} (2k+1)^2$. (5%)

5. A tetrahedron has its center at point O and four apexes at A, B, C and D. Find the value of the cosine of the angle subtended by $\overrightarrow{OA}$ and $\overrightarrow{OB}$. (10%)

6. (a) Find the volume enclosed by the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ (5%)

(b) Find the surface area of this ellipsoid. (5%)

7. Function $f(x, y, z)$ is a homogeneous function of first degree,

$$f(\lambda x, \lambda y, \lambda z) = \lambda f(x, y, z),$$

and $f(x_0, y_0, z_0) = 1$.

(a) Find the value of

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z}$$

at $(x, y, z) = (x_0, y_0, z_0)$. (10%)

(b) Find the value of

$$x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + z^2\frac{\partial^2 f}{\partial z^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + 2yz\frac{\partial^2 f}{\partial y \partial z} + 2zx\frac{\partial^2 f}{\partial z \partial x}$$

at $(x, y, z) = (x_0, y_0, z_0)$. (5%)