國立交通大學 97 學年度碩士班考試入學試題

科目：工程數學(3041)　　一般、在職　　考試日期：97 年 3 月 8 日 第 1 節

系所班別：機械工程學系　　組別：機械系丁組　　第 / 頁，共 / 頁

*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

Prob. 1 (16 %)

(a) Find the Fourier transform $X(j\omega), j=\sqrt{-1}$ of the following function:

$$x(t)=\begin{cases}1/T, & -T/2\le t\le T/2\\ 0, & \text{otherwise}\end{cases}$$

where T is a positive constant.

(b) What happens to $x(t)$ and $X(j\omega)$ if $T\to 0$?

Prob. 2 (17 %)

In the two-dimensional Cartesian coordinates, what is the shortest and the longest distances from the origin to the curve $5x^2+6xy+5y^2=8$?

Prob. 3 (17 %)

Solve the following PDE for $u(x,t)$:

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

st.

$$u(0,t)=u(L,t)=0, u(x,0)=\sin\left(\frac{2\pi x}{L}\right),\ \frac{\partial u}{\partial t}(x,0)=0,$$

where c and L are positive constants.

Prob. 4 (16 %)

Solve the differential equation

$y'-2xy=0$

Prob. 5 (17 %)

Find the eigenvalues and the corresponding eigenvectors for

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

[Hint: one of the eigenvalues is 5]

Prob. 6 (17 %)

State and prove the Existence theorem for Laplace transformation.