**考生請注意：必須寫出作答過程或得到答案之理由，只寫答案不予計分。**

1. Let $a$ and $n$ be integers. Show: If $a \mid n$ and $a \mid (n+2)$, then $a \mid 2$. (10%)

2. Show: For all $n \in Z$. $(n^3 - n) \bmod 3 = 0$. (10%)

3. If a vending machine dispenses only 3-cent and 8-cent stamps, then any monetary value of 14 cents or greater can be obtained from this machine.
   (a) How could 20 cents be obtained? (5%)
   (b) Prove the general result. Hint: by Principle of Mathematical Induction. (15%)

4. Draw a Hasse diagram for the "divides" relation $\mid$ on the set of positive divisors of 30. (10%)

5. Let V={1, 2, 3, 4}, E={{1, 4}, {2, 4}, {3, 4}}, and graph G=(V, E).
   (a) Draw G=(V, E). (5%)
   (b) Determine the edge set for the subgraph of G induced by W={1, 2, 3}. (5%)
   (c) What is the distance from 2 to 3? (5%)
   (d) Is G bipartite? (5%)

6. (a) Draw the graph $K_{4,4}$. (5%)
   (b) Find a Hamiltonian cycle in $K_{4,4}$.(Draw it out.) (5%)

7. (a) How many functions are there from an $n$-element set onto {0, 1}? (5%)
   (b) How many strings can be formed by ordering the letters *PARAGRAPH* ? (5%)

8. Show two partitions of 15 and draw their corresponding Ferrers graphs. (10%)