Problem 1 (30%) Figure 1 shows a circuit to be controlled, where current u is the control input and voltage y is the output.

(a)(10%) Find the transfer function of the circuit.

(b)(10%) Estimate resistance R and capacitance C from the measured output waveform y in Fig. 2 when the input u is a unit step function.

(c)(10%) Suppose that we want to regulate the output y to 1 V and thus devise a control circuit as shown in Fig. 3 with the reference voltage r being set to 1 V. Please draw the corresponding block diagram of the circuit in Fig. 3 and choose from the following list of types of control it belongs to (multiple choices): feedforward control, feedback control, P control, I control, D control, PI control, PID control.

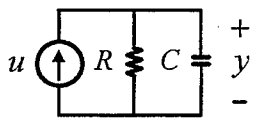

Fig. 1

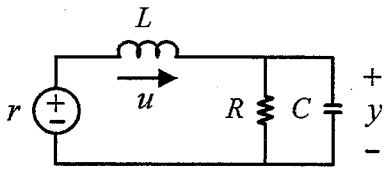

Fig. 3

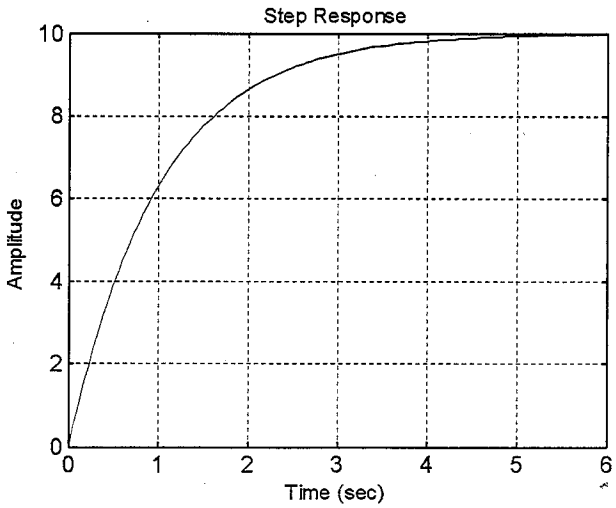

Fig. 2

Problem 2 (20%) A simple model of a car in motion is shown in Fig. 4, where the driving force f is 100 N, the air drag and friction is modeled as a linear damper with a damping coefficient b=10 N·s/m, and the car weighs m=100 kg.

(a)(10%) Derive a state-space model of the system with the car speed v as an output and the driving force f as an input.

(b)(10%) Determine the car speed using the state transition matrix under the assumption of zero initial state.

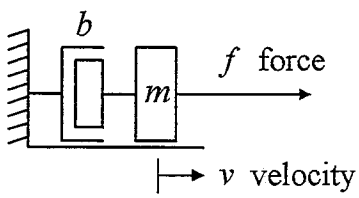

Fig. 4

Problem 3 (20%) Figure 6 shows the Bode diagram of the op-amp of Fig. 5.
(a) (10%) Given a sinusoidal input $u(t)=\sin(30t)$, estimate the steady state output $y(t)$.
(b) (10%) Use this op-amp to build a non-inverting amplifier as shown in Fig. 7. Is it stable? Estimate the phase margin of the resulting non-inverting amplifier.

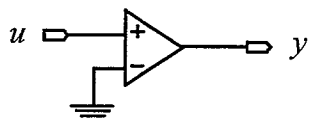

Fig. 5

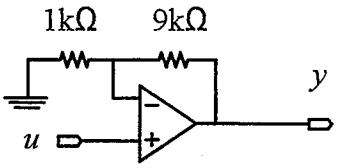

Fig. 7

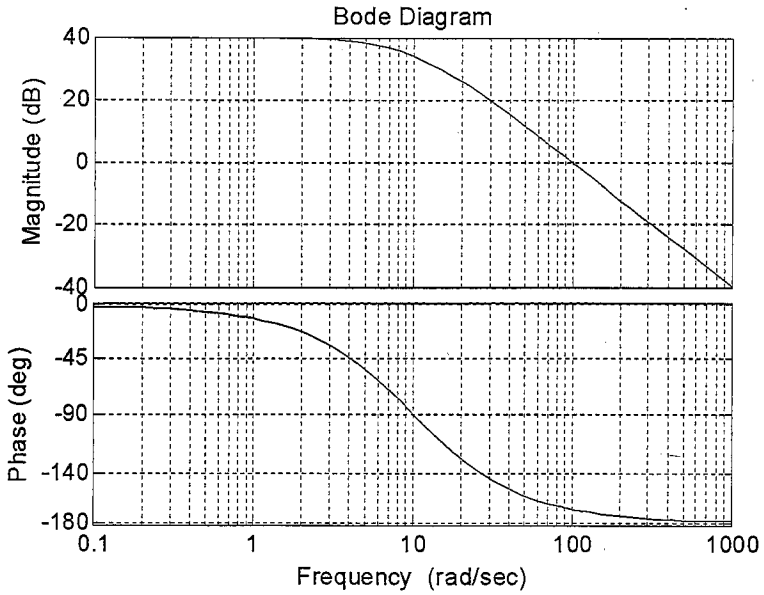

Fig. 6

Problem 4 (15%) Given a feedback system shown in Fig. 8.
(a) (5%) Determine the value of K to satisfy a damping ratio of 1.
(b) (10%) With K=1 and r =1, find the steady state regulation error e.

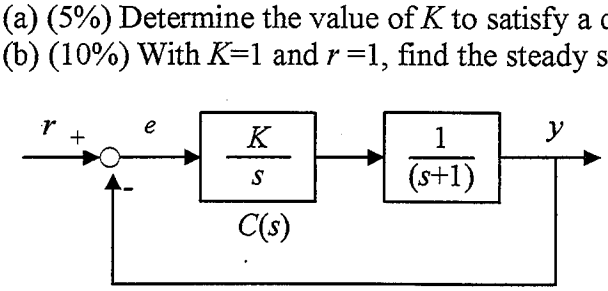

Fig. 8

Problem 5 (15%) A system is described by the following input-output relationship.

$$y(t)=\int_0^t e^{-2\tau}\sin(\tau)u(t-\tau)d\tau\,.$$

(a) (5%) Is it a linear system? Justify your answer.
(b) (10%) Determine the poles of the system.