朝陽科技大學 97 學年度碩士班招生考試試題

系（所）別：資訊工程系　　總分：100 分
組　　別：一般生甲組
科　　目：離散數學　　

1. Using (a) permutation and (b) set concept to compute the number of (staircase) paths in the *xy*-plane from (2, 2) to (7, 6), where each such path is made up of individual steps going one unit to the right (R) or one unit upward (U). (For example, the following path shown in Fig. 1 is a staircase path from (2,1) to (7,4)) (10%)

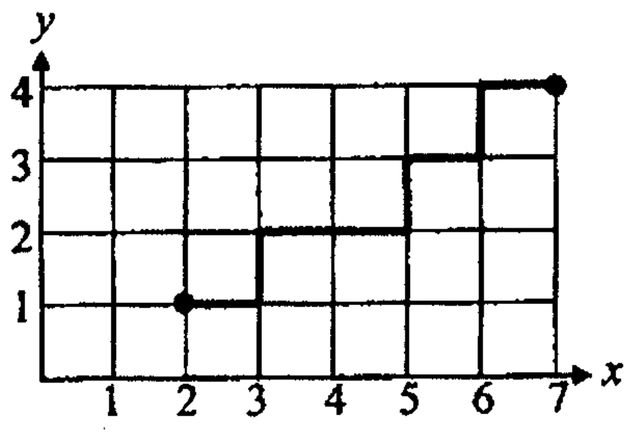

Fig. 1

2. Please answer the following questions.
   (a) What is the number of arrangements of the letters in TALLAHASSEE? (5%)
   (b) How many of these arrangements have no adjacent L's? (5%)

3. Let p, q, r be three primitive statements. Please verify that $[p\rightarrow(q\rightarrow r)] \rightarrow [(p\rightarrow q)\rightarrow(p\rightarrow r)]$ is a tautology. That is, verify that $[p\rightarrow(q\rightarrow r)] \rightarrow [(p\rightarrow q)\rightarrow(p\rightarrow r)]$ is true in any case. (10%)

4. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$, where $n \geq 2$ and $a_0 = -1$, $a_1 = 8$. (10%)

5. Please answer the following questions.
   (a) Find a minimal spanning tree in Fig. 2. (5%)
   (b) Is the minimal spanning tree in part (a) unique? (5%)

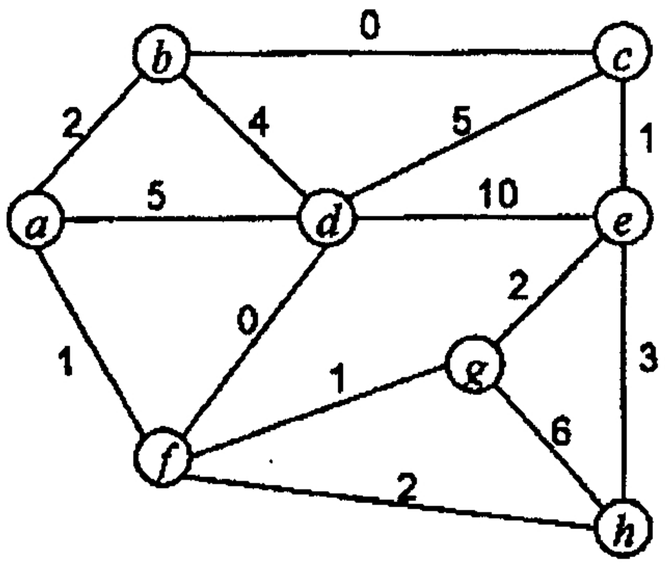

Fig. 2

6. A student is to answer ten out of 14 questions on an examination. In how many ways can he make his selection if (a) there are no restrictions? (b) he must answer the first two questions? (c) he must answer at least four of the first seven questions? (10%)

7. In how many ways can eight people, denoted A, B, ..., H be seated about the square table shown in Fig. 3, where Figs. 3(a) and 3(b) are considered the same but are distinct from Fig. 3(c)? (10%)

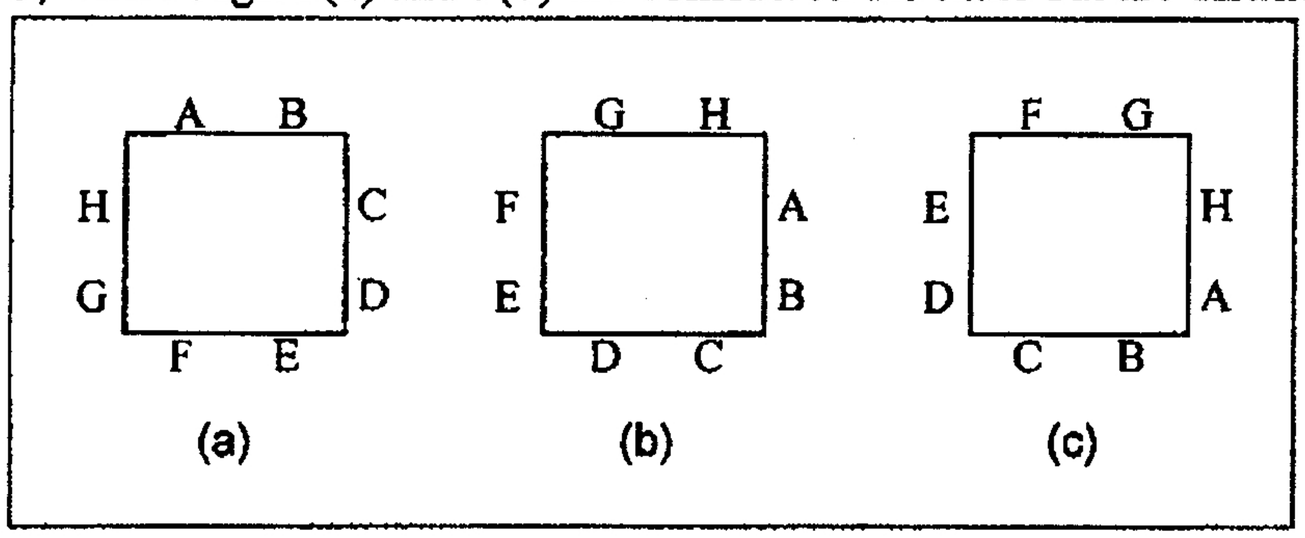

Fig. 3

8. Prove that $1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(2n-1)(2n+1)}{3}$ for all $n \geq 1$ by the Principle of Mathematical Induction. (10%)

9. Please answer the following questions.
(a) We would like to construct roads for 5 villages such that no village is isolated. How many ways can we do this? (5%)
(b) Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3, or 5. (5%)

10. The *Fibonacci number* is defined recursively by (1) $F_0 = 0$, $F_1 = 1$; and (2) $F_n = F_{n-1} + F_{n-2}$, for $n \in Z^+$ with $n \geq 2$. Please answer the following questions.
(a) For $\alpha = (1+\sqrt{5})/2$, show that $\alpha^2 = \alpha+1$. (3%)
(b) If $n \in Z^+$, prove that $\alpha^n = \alpha F_n + F_{n-1}$, where F_n is the n-th Fibonacci number. (7%)