

國立嘉義大學九十七學年度
光電暨固態電子研究所碩士班招生考試試題

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科目：工程數學

Write only either "True" or "False", in answering the following true-false questions 1-10
(1-10 題寫下 "True" 或是 "False". 一題 4 分，答錯倒扣 2 分。)

1. $x(t) = e^{\beta t}(c_1 \cos \omega t + c_2 \sin \omega t)$ is the general solution of the ordinary differential equation $\frac{d^2x}{dt^2} - 2\beta \frac{dx}{dt} + (\omega^2 + \beta^2)x = 0$, where c_1 and c_2 are arbitrary numbers. True or false?

2. $y(x) = \cosh^{-1} x$ is a particular solution of the ordinary differential equation $(x^2 + 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$. True or false?

3. If vectors \bar{a} , \bar{b} , and \bar{c} are not coplanar and we have $\bar{a}' = \frac{\bar{b} \times \bar{c}}{\bar{a} \cdot (\bar{b} \times \bar{c})}$, then $\bar{a}' \cdot \bar{b} = \bar{a}' \cdot \bar{c}$ holds. True or false?

4. If vectors \bar{a} , \bar{b} , and \bar{c} are not coplanar and we have $\bar{a}' = \frac{\bar{b} \times \bar{c}}{\bar{a} \cdot (\bar{b} \times \bar{c})}$, and $\bar{b}' = \frac{\bar{c} \times \bar{a}}{\bar{a} \cdot (\bar{b} \times \bar{c})}$, then $\bar{a}' \cdot \bar{a} = \bar{b}' \cdot \bar{b}$ holds. True or false?

5. If a matrix A is orthogonal (i.e., $A^T = A^{-1}$), the determinant must be $|A| = -1$. True or false?

6. The matrix $A = \begin{bmatrix} \sqrt{3} & -\sqrt{2} & -\sqrt{3} \\ 1 & \sqrt{6} & -1 \\ 2 & 0 & 2 \end{bmatrix}$ is Hermitian. True or false?

7. If the inner product of two functions is defined as $\langle f|g \rangle = \int_{-\pi}^{\pi} f^*(x)g(x)dx$ (where * denotes the complex conjugate), then f and g are said to be orthogonal to each other when $\langle f|g \rangle = 0$. True or false?

8. If the inner product of two functions is defined as $\langle f|g \rangle = \int_{-\pi}^{\pi} f^*(x)g(x)dx$ (where * denotes the complex conjugate), then we must have $\langle f|f \rangle = 1$ for any function f . True or false?

9. If the Fourier transform $\tilde{f}(k)$ of any function $f(x)$ is defined as $\tilde{f}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)e^{-ikx}dx$, and δ denotes the Dirac delta function, then the Fourier transform of $\sin k_0 x$ is given by $\sqrt{2\pi} \delta(k - k_0)$. True or false?

10. If the Fourier transform $\tilde{f}(k)$ of any function $f(x)$ is defined as $\tilde{f}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)e^{-ikx}dx$, and δ denotes the Dirac delta function, then the Fourier transform of $\cos k_0 x$ is given by $\frac{\sqrt{2\pi}}{2} [\delta(k - k_0) + \delta(k + k_0)]$. True or false?

(11-13 題為計算題，必須寫下計算過程及必要之說明。每題 20 分)

11. The falling parachutist (mass m) encounters the air resistance $-bv^2$, opposing the force of the gravitational attraction, mg , of the earth. Initial condition is given by $v=0$ at time $t=0$. Solve Newton's equation of motion $m \frac{dv}{dt} = mg - bv^2$. Find the velocity as a function of time, in terms of $v_0 = \sqrt{mg/b}$ and $T = \sqrt{m/(gb)}$.

12. The action of the control mechanism on a particular system for an input $f(t)$ is described, for $t \geq 0$, by the coupled first-order equations $\frac{dy}{dt} + 4z = f(t) = H(t)$, $\frac{dz}{dt} - 2z = \frac{dy}{dt} + \frac{1}{2}y$, where H denotes the Heaviside unit step function. Find the response $y(t)$ of the system, given that $y(0) = 1$ and $z(0) = 0$.

13. Find the complex Fourier expansion of $f(x)$, which is given by

$$f(x) = \sum_{n=1}^{\infty} \frac{-(-1)^n}{n\pi} \sin(2n\pi x) = \frac{1}{\pi} \sin 2\pi x - \frac{1}{2\pi} \sin 4\pi x + \frac{1}{3\pi} \sin 6\pi x - \dots$$