

國立嘉義大學九十七學年度

土木與水資源工程學系碩士班招生考試試題

科目：工程數學 甲 乙 組

說明：如條件不足，請自行假設。

1. Solve the following ordinary differential equation: (20%)

$$y'' + y' - 2y = 6e^x$$

2. Solve the following ordinary differential equations using Laplace Transformation: (20%)

① $y(t) = 1 + \int_0^t y(\tau) d\tau$

② $y(t) = 1 + y'(t)$, $y(0) = 1$

3. Solve the following ordinary differential equation: (20%)

$$y^{(4)} + 3y = e^{2x}$$

4. Define $\nabla = \frac{\partial}{\partial x} \bar{i} + \frac{\partial}{\partial y} \bar{j} + \frac{\partial}{\partial z} \bar{k}$ as an operator, a function $F(x, y, z) = x^2 + y^2 + z^2$,

and a vector $\bar{v} = x^3 \bar{i} + y^3 \bar{j} + z^3 \bar{k}$ are known. Calculate results of the following questions: (20%)

① gradient $\nabla F = ?$ ② divergence $\nabla \cdot \bar{v} = ?$ ③ $\text{curl } \nabla \times \bar{v} = ?$

④ $\nabla \nabla F = ?$ ⑤ $\nabla \cdot \nabla F = ?$ ⑥ $\nabla \times \nabla F = ?$

⑦ $\nabla \nabla \bar{v} = ?$ ⑧ $\nabla \cdot \nabla \cdot \bar{v} = ?$ ⑨ $\nabla \times \nabla \times \bar{v} = ?$ ⑩ $\nabla \cdot \nabla \times \bar{v} = ?$

5. For the following matrix, (a) find the eigenvalues and eigenvectors;

(b) diagonalize the matrix A by using eigenvectors. (20%)

$$A = \begin{bmatrix} -3 & 2 \\ -10 & 6 \end{bmatrix}$$