# 國立嘉義大學九十七學年度
# 生物機電工程學系碩士班招生考試試題

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**科目：工程數學**　　　　　　　　（※禁止使用計算機）

1. A function $y = y(x)$ satisfies the $1^{st}$-order differential equation:

$$2xy\frac{dy}{dx} + 4x + 3y^2 = 0$$

(a) Does the differential equation satisfy the "Condition of Exactness" ? (10%)
(b) Solve the differential equation using the method of integration factor. (10%)

2. Consider the differential equation $\rho\frac{\partial u}{\partial t} = -\sum_{n=0}^{N} C_n e^{in\omega t} + \mu(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r})$, where $n$, $\rho$, $\mu$, $C_n$, and $\omega$ are arbitrary constants, $\sqrt{-1} = i$, and $u(r,t) = \sum_{n=0}^{N} v_n(r)e^{in\omega t}$.

(a) Reduce the equation given above to Bessel's equation. (10%)
(b) The general solution of the Bessel's equation given in part (a) is

$$v_n(r) = A_n J_0(\beta\frac{r}{a}n^{1/2}i^{3/2}) + B_n Y_0(\beta\frac{r}{a}n^{1/2}i^{3/2}) + \frac{iC_n}{\rho n\omega}$$

where $A_n$, $B_n$, $a$, and $\beta$ are arbitrary constants. The boundary conditions are

$$v_n = 0 \quad when \quad r = a,$$

$$\frac{\partial v_n}{\partial r} = 0 \quad when \quad r = 0.$$

Find $A_n$ and $B_n$. (10%)

3. Consider the following linear equation with 3 unknowns $x$, $y$ and $z$:

$$\begin{cases} ay + z = b \\ ax + bz = 1 \\ ax + ay + 2z = 2 \end{cases}$$

Determine constants $a$ and $b$ (if any) such that the system possesses the following:
(a) A unique solution. (5%)
(b) A one-parameter solution. (5%)
(c) A two-parameter solution. (5%)
(d) No solution. (5%)

4. Evaluate the followings:

(a) the Laplace transform of the piecewise function $f(t) = \begin{cases} 2, & 0 \le t < 3 \\ -2, & t \ge 3 \end{cases}$. (7%)

(b) the inverse Laplace transform of $G(s) = \frac{1}{s^2(s^2+b^2)}$. (6%)

(c) the inverse Laplace transform of $H(s) = \ln\frac{s^2+b^2}{s^2}$. (7%)

5. Given a matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, construct an orthogonal matrix $R$ from $A$. (20%)