

國立嘉義大學九十七學年度  
應用數學系碩士班招生考試試題

科目：線性代數

說明：本考試試題為計算、證明題，請標明題號，同時將過程作答在「答案卷」上。

(1~4 題每題 10 分，5~8 題每題 15 分，共 100 分)

1. To find a matrix  $A \in \mathbb{R}^{3 \times 3}$  such that for all  $x, y, z \in \mathbb{R}$ ,

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x - 2y + 3z = 0 \right\}. \quad (10\%)$$

2. Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation given by  $T(x, y) = (x + 3y, 2x + y)$ . Compute the adjoint  $T^*$  of  $T$ . Is  $T$  self-adjoint? (10%)

3. Prove that  $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  are not similar. (10%)

4. Let  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & a \end{bmatrix}$ , where  $a \in \mathbb{R}$ . Prove that there exists uniquely one value of  $a$  such that  $\det(A) = 0$ . (10%)

5. Let  $n$  be an integer and let  $t_0, t_1, \dots, t_n$  be distinct real numbers. Show that the set  $\left\{ \phi_j(x) = \prod_{i=0, i \neq j}^n \frac{x - t_i}{t_j - t_i} \mid j = 0, 1, \dots, n \right\}$  is a basis for  $P_n(\mathbb{R})$ . (15%)

6. Find the Fourier coefficients of the vector  $x^2 + x + 1$  with respect to the orthogonal set of vectors  $\left\{ 1, x, x^2 - \frac{1}{3} \right\}$ . (15%)

7. Let  $A = \begin{bmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & a_2 & 0 \\ 0 & a_3 & 0 & 0 \\ a_4 & 0 & 0 & 0 \end{bmatrix}$ , where  $a_1, a_2, a_3, a_4 \in \mathbb{R}$  and let  $a_1 a_4 > 0$  and  $a_2 a_3 > 0$ . Show that  $A$  is diagonalizable. (15%)

8. Find the Jordan canonical forms of  $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  and  $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ . (15%)