

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：微積分【應數系碩士班乙組】

題號：4051
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共 8 題。答題時，每題都必須寫下題號與詳細步驟。

[1]. (10%) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\tan x} \right)$.

[2]. (12%) Let $f(x) = 2e^{g(x)}$ and $g(x) = \int_4^{x^2} \frac{2t}{1+t^4} dt$. Compute $f'(2)$.

[3]. Solve the following differential equations.

(a) (10%) $xy' = 2y + x^3 \sin x$, $y(\pi) = 0$.

(b) (10%) $(2x^2 + y^2) dx + xy dy = 0$, $y(1) = 0$.

[4]. (12%) Evaluate the integral $\int_1^{\infty} (1-x)e^{-x} dx$.

[5]. (12%) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n5^{n+1}}$.

[6]. (10%) Find the Maclaurin series for $f(x) = x^3 \sin\left(\frac{x}{2}\right)$.

[7]. (12%) Evaluate the integral $\int_0^1 \int_{4y^2}^4 e^{\sqrt{x}} dx dy$.

[8]. (12%) Show that the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ in the coordinates (r, θ) ,

where $x = r^2 \cos \theta$ and $y = r^2 \sin \theta$ is $\frac{1}{r^4} \frac{\partial^2 f}{\partial \theta^2} + \frac{1}{4r^3} \frac{\partial f}{\partial r} + \frac{1}{4r^2} \frac{\partial^2 f}{\partial r^2} = 0$.

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