國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：數值分析【應數系碩士班乙組】

題號：4053

Please write down all the detail of your computation and answers.

1. (20%) Use (1) Lagrange formula, (2) Neville's method, and (3) Newton divided difference formula to compute the cubic polynomial $p(x)$ interpolating the following data

x	-1	0	1	2
y	4	3	2	7

2. (20%) (1) State the secant method. Newton method and the fixed point method to find a root of a given nonlinear equation. (2) State the advantage, disadvantage and the order of convergence of each method.

3. (20%) Derive the midpoint rule and composite midpoint rule for numerical integration with error formula.

4. (20%) Assume the Gaussian elimination on $n \times n$ matrix A needs no row exchange. Write a program to compute the LU (triangular) factorization of A using the least memory. How much storage is needed to run your program?

5. (20%) Let A be an $n \times n$ nonsingular matrix, $\mathbf{x}$ and $\hat{\mathbf{x}}$ be the exact and numerical solutions of linear system $A\mathbf{x} = \mathbf{b}$ respectively, and the residual $\mathbf{r} = \mathbf{b} - A\hat{\mathbf{x}}$. Show that

$$\frac{1}{\kappa(A)}\frac{||\mathbf{r}||}{||\mathbf{b}||} \leq \frac{||\hat{\mathbf{x}} - \mathbf{x}||}{||\mathbf{x}||} \leq \kappa(A)\frac{||\mathbf{r}||}{||\mathbf{b}||},$$

where $\kappa(A)$ is the condition number of A. Interpret the meaning of this error analysis and explain why it is important.