國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：數理統計【應數系碩士班甲組】

題號：4052
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共十題，每題 10 分。答題時，每題都必須寫下題號與詳細步驟。
請依題號順序作答，不會作答題目請寫下題號並留空白。

1. If X is an exponential random variable with mean $\frac{1}{\lambda}$, find that $E[X^k]$, $k = 1, 2, \ldots$

2. Find the probability density function of $Y = e^X$ when X is normally distributed with parameters μ and σ^2.

3. Let X_1, X_2, and X_3 be uncorrelated random variables, each with mean μ and variance σ^2. Find, in terms of μ and σ^2, $\mathrm{Cov}((X_1 + X_2)(X_2 + X_3))$ and $\mathrm{Cov}((X_1 + X_2)(X_1 - X_2))$

4. Let $X_1, \ldots, X_n$ be a random sample from a population with pdf

$$f_X(x) = \begin{cases} 1/\theta & 0 < x < \theta \\ 0 & \text{otherwise.} \end{cases}$$

Let $X_{(1)} < \cdots < X_{(n)}$ be the order statistics. Show that $X_{(1)}/X_{(n)}$ and $X_{(n)}$ are independent random variables.

5. Given that $N = n$, the conditional distribution of Y is χ^2_{2n}. The unconditional distribution of N is Poisson(θ). Calculate $E[Y]$ and $\mathrm{Var}(Y)$.

6. Let $X_1, \ldots, X_n$ be a random sample from the pdf

$$f(x|\mu,\sigma) = \frac{1}{\sigma}e^{-(x-\mu)/\sigma}, \quad \mu < x < \infty,\ 0 < \sigma < \infty.$$

Find a two-dimensional sufficient statistic for (μ, σ).

7. Let $X_1, \ldots, X_n$ be a random sample from a population with pdf

$$f(x|\theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \theta > 0.$$

   (a) Is ΣX_i sufficient for θ?
   (b) Find a complete sufficient statistic for θ.

8. Let $X_1, \ldots, X_n$ be a random sample from the pdf

$$f(x|\theta) = \theta x^{-2}, \quad 0 < \theta \le x < \infty.$$

   (a) Find the MLE of θ.
   (b) Find the method of moments estimator of θ.

9. Suppose that we have two independent random samples: $X_1, \ldots, X_n$ are exponential(θ), and $Y_1, \ldots, Y_m$ are exponential(μ). Find the likely ratio test (LRT) of $H_0\colon \theta = \mu$ versus $H_1\colon \theta \neq \mu$.

10. Derive a confidence interval for a binomial p by inverting the LRT of $H_0\colon p = p_0$ versus $H_1\colon p \neq p_0$.