國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：基礎數學【應數系碩士班甲組】

答題時，每題須寫下題號與詳細步驟。請依題號順序作答，不會作答題目請寫下題號並留空白。

題目紙上的答案不予計分。

(15%)1. Find all the positive values of p for which the following series converges (You have to give the reasoning).

$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^p}$$

(10%)2. What is the nullspace for the following matrix?

$$\begin{pmatrix} 1 & 1 & 0 & -3 \\ 2 & 2 & 1 & -10 \\ 0 & 0 & 1 & -4 \\ 4 & 4 & 3 & -24 \end{pmatrix}$$

(15%)3. $f(x)$ is defined in $(-\pi, \pi)$ as follows:

$$f(x) = \begin{cases} x^{1.2}\sin(1/x) + |x^3 - x| & \text{if } x \in (-\pi, 0) \text{ or } x \in (0, \pi); \\ 0 & \text{if } x = 0. \end{cases}$$

Find all the values of x at which f is differentiable. (You have to give the reasoning).

4. Define matrix A as follows:

$$\begin{pmatrix} 3 & \sqrt{6} \\ \sqrt{6} & 4 \end{pmatrix}$$

Answer the following two questions:

(15%)(a) Assume A can be expressed as KDK^t, where D is a diagonal matrix and K^t is the transpose of matrix K. Find K and D.

(15%)(b) Calculate $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp^{-(3x^2+2\sqrt{6}xy+4y^2)}\, dxdy$.

(10%)5. $f(x, y)$ is defined as follows:

$$f(x, y) = \int_{x^2+1}^{2} \frac{1}{1 + y^2 t^4} dt.$$

Calculate $\frac{\partial f}{\partial x}$.

(20%)6. Find the maximum of $f(x, y) = e^{xy}$ when x and y are both positive and constrained by the equation $x^2 + y^2 = 8$.