國立嘉義大學九十七學年度
數學教育研究所碩士班（一般生）招生考試試題

科目：微積分

1. Let the function $f(x)=\begin{cases} x^2-x, & \text{if } x \text{ is irrational} \\ m\,x+b, & \text{if } x \text{ is rational} \end{cases}$, where m and b are constants. Determine the constants m and b such that f is continuous and differentiable at 1. (10%)

2. Find the limit: (a). $\lim_{x\to 0}\frac{1}{x}\int_0^{x^2}\sqrt{t}\sin t\,dt$. (10%)

   (b). $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$. (10%)

3. Find the extreme values of the function $f(x,y)=x^2+y^2-2x+2y+3$ subject to the constraint $x^2+y^2\le 8$. (20%)

4. Evaluate $\int_0^{\infty}\frac{x^2}{(x^2+1)^2}\,dx$. (10%)

5. Find the shortest distance from the point $(-1,1,1)$ to the set of points given by the equation $z=xy$. (20%)

6. Let $f(x)=\frac{3}{5-2x}$. (20%)

   (a) Write $f(x)$ as a power series centered at 0, i.e., $f(x)=\sum_{n=0}^{\infty}a_n x^n$.

   (b) Determine the interval of convergence of this power series.