

國立高雄第一科技大學 97 學年度 碩士班 招生考試 試題紙

系所別：風險管理與保險系

組別：精算資訊組

考科代碼：4221

考科：統計學

注意事項：

- 1、本科目可使用本校提供之電子計算器。
- 2、請於答案卷上規定之範圍作答，違者該題不予計分。

請依題號順序作答，否則不予計分

第一部份：共 7 題，每題 8 分。

1. A pair of fair dice is tossed 6 times in succession. What is the probability of observing no 4's and no 7's in any of the 6 tosses?
2. A sample of size 2 is drawn at random and without replacement from the population  $\{1, 2, 3, 4, 5\}$ . What is the probability that the range of the sample is equal to 2?
3. In rolling a pair of fair dice, what is the probability that a sum of 7 is rolled before a sum of 9 is rolled?
4. A die was rolled 30 times with the results shown below.

Number of Spots	1	2	3	4	5	6
Frequency	2	3	10	8	3	4

If a chi-square goodness-of-fit test is used to test the hypothesis that the die is fair, then what is the value of the chi-square statistic?

5. A card hand selected from a standard card deck consists of 2 kings, 2 queens, and a jack. Three additional cards are selected at random and without replacement from the remaining cards in the deck. What is the probability that the enlarged hand contains at least three kings?
6. Let  $X$  and  $Y$  be random variables with variances 2 and 3, respectively, and covariance  $-1$ . Let  $W = aX + (1-a)Y$ , where  $a$  is a constant. What is the value of  $a$  such that the random variable  $W$  has the smallest variance?
7. Suppose the straight line of the form  $y = bx + 2$  is fitted to the five data points  $(0, 3)$ ,  $(1, 3)$ ,  $(2, 1)$ ,  $(3, 0)$  and  $(4, 0)$  by the method of least squares. What is  $b$ ?

第二部份：共 4 題，每題 11 分。

1. Let  $X$  and  $Y$  be continuous random variables with joint density function

$$f(x, y) = \begin{cases} 8xy & \text{for } 0 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

and let  $W = XY$ . What is  $P\left(W < \frac{1}{2}\right)$ ?

2. Let  $X_1, X_2, X_3$ , and  $X_4$  be a random sample from the discrete distribution  $X$  such that  $P(X = x) = e^{-\theta^2} \frac{\theta^{2x}}{x!}$  for  $x = 0, 1, 2, \dots$ , and 0 otherwise, where  $\theta > 0$ . If the data are 40, 18, 33, and 9, what is the maximum likelihood estimate of  $\theta$ ?
3. Let  $X_1, \dots, X_5$  and  $Y_1, \dots, Y_8$  be independent random samples from normal distributions with common mean  $\mu = 10$  and common variance  $\sigma^2 > 0$ . The statistic  $W = k \frac{(\bar{X} - 10)^2}{(\bar{Y} - 10)^2}$  has an  $F$  distribution with degrees of freedom 1 and 1. Determine  $k$ .

第 2 頁，合計 2 頁【尚有試題】

4. Let  $X$  and  $Y$  be continuous random variables with joint density function

$$f(x, y) = \begin{cases} \frac{12}{25}(x + y^2) & \text{for } 1 < x < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the marginal density function of  $X$  where nonzero?