國立高雄第一科技大學 97 學年度 碩士班 招生考試 試題紙

系所別：系統資訊與控制研究所　　組別：控制組
考科代碼：1421　　考科：工程數學

注意事項：
1、本科目可使用本校提供之電子計算器。
2、請於答案卷上規定之範圍作答，違者該題不予計分。

1 A linear differential equation

$$\frac{dy}{dt} + 4y = f(t)$$

is found to have a particular solution
$y = 2t^2 - t + 1$ when $f(t) = 8t^2 + 3$ and a particular solution
$y = 0.8\cos(3t) + 0.6\sin(3t)$, when $f(t) = 5\cos(3t)$,

a) Suggest a particular solution when $f(t) = 10\cos(3t) + 4t^2 + (3/2)$ (3%), and show by substitution that your solution is correct. (2%)

b) Suggest a particular solution when $f(t) = 5\cos(3(t-10))$ (3%), and show by substitution that your solution is correct. (2%)

2 Solve the following differential equations with the given initial conditions

(a) $\frac{dy}{dx} - 3y = 0$, $y(0) = 1$; (10%)

(b) $\frac{dy}{dt} + 5 = \sin(12t)$, $y(0) = 0$; (10%)

(c) $3\frac{dy}{dt} + 2y = e^{-t}$, $y(0) = 3$. (10%)

3 An LRC circuit as shown in Fig. 1 obeys the equation
$Ld^2/dt^2 + Rdq/dt + q/C = v(t)$
Where q is the charge on the capacitor, v(t) is the applied voltage, L is the inductance, R is the resistance, and C is the capacitance. Find a steady state solution for q as $R = 120\Omega$, $L = 0.06H$, $C = 0.0001F$, $v(t) = 130\cos(1000t)$, and hence calculate the voltages across the capacitor, resistor, and inductor, given by $v_C = q/C$, $v_R = Rdq/dt$ and $v_L = Ld^2q/dt^2$. (10%)

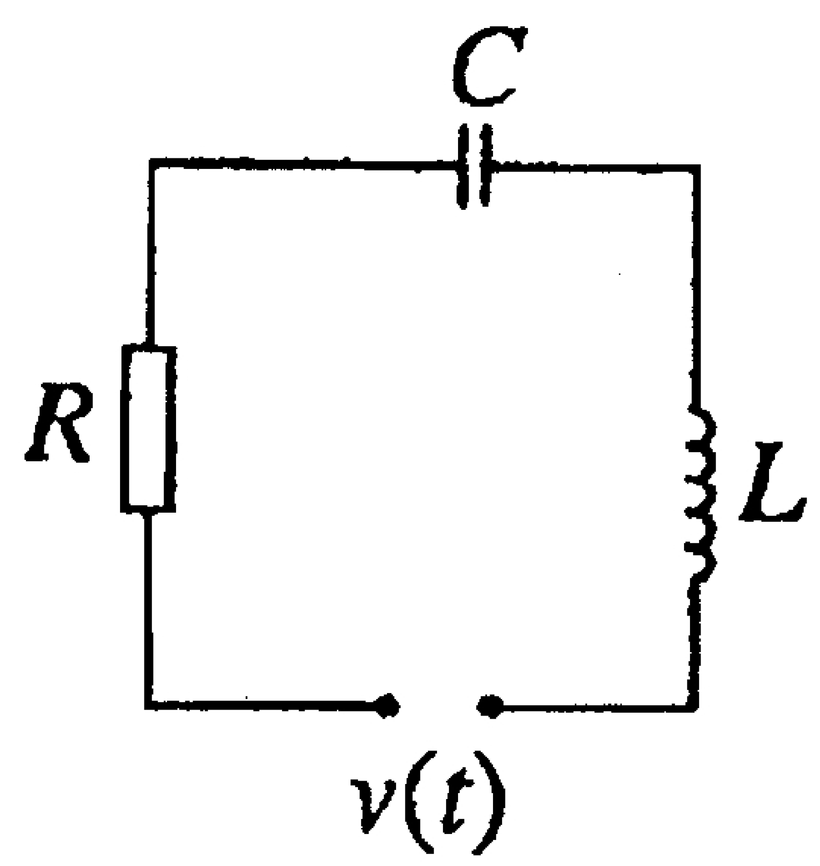

Fig. 1. A LRC circuit.

4 An RC circuit is subjected to a single frequency input of angular frequency w and magnitude v_i. Find the steady state solution of the equation
$Rdq/dt + (q/C) = v_i e^{jwt}$, (6%)
and hence find the voltages across capacitor ($v_C = q/C$) (2%), and resistor ($V_R = Rdq/dt$). (2%)

5 Evaluate the integral $\int_C \frac{\sin z}{(z-z_0)^2} dz$, where C is a simple closed curve for the following cases (i) z_0 is not enclosed by C (ii) z_0 is enclosed by C. (10%)

6 Solve the differential equation
$\frac{dy}{dt} + y = \delta(t-a)$
with $y(0) = 1$, $a > 0$,
(i) by the Laplace transform method, (5%)
(ii) by finding the integrating factor. (5%)

7 Solve the following exact first-order differential equations (10%)
(i) $xdx + ydy = (x^2 + y^2)dy$
(ii) $ye^{xy}dx + xe^{xy}dy = 0$

8 Solve the following separable first-order differential equations (10%)
(i) $x^2dx + 3y^3dy = 0$
(ii) $xydx + \sqrt{1-x^2}\,dy = 0$