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10. The production level P of a factory during one time period is modeled by $P(x,y) = Kx^{1/2}y^{1/2}$ where K is a positive integer, x is the number of units of labor scheduled and y is the number of units of capital invested. If labor costs \$2700/unit, capital cost \$600/unit and the owner has \$1,600,000 available for one time period, what amount of labor and capital would maximize production?

    (a) 296.3 units of labor and 1333.3 units of capital

    (b) 592.6 units of labor and 2666.7 units of capital

    (c) 2666.7 units of labor and 592.6 units of capital

    (d) 1333.3 units of labor and 296.3 units of capital

Section III. Problems (計算與證明題, 每題10分, 請清楚寫出計算或證明過程, 否則不予計分)

1. Suppose that the function f is continuous on $[a,b]$ and differentiable on (a,b). Prove that if $f'(x) \neq 0$ for any x in (a,b), then there exists at most one value $c \in [a,b]$ such that $f(c) = 0$. (10%)

2. (a) Suppose that $\{a_n\}$ is a sequence of nonnegative real numbers. Prove that $\sum a_k$ is convergent if and only if its sequence of partial sums is bounded. (5%)

   (b) Give an example of a series that diverges and whose sequence of partial sums is bounded. (5%)

3. Assume that the rectangle $R = [0,1] \times [0,1]$ and that f is defined on R by

$$f(x,y) = \begin{cases} \dfrac{x-y}{(x+y)^3} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   Show that f is not Riemann integrable on R. (10%)

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Section I. True/False (是非題, 每題3分, 不需說明理由)

1. If $\{a_n\}$ and $\{b_n\}$ both diverge, then $\{a_n b_n\}$ diverges.
2. If the function f is defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational,} \end{cases}$$

then $\lim_{x\to 0} f(x) = 0$.

3. If $\lim_{x\to\infty} |f(x)| = |L|$, then $\lim_{x\to\infty} f(x) = L$.
4. A function f is continuous at $x = a$ if and only if $\lim_{x\to a} f(x) = f(a)$.
5. A composition of two continuous functions might **not** be continuous.
6. Limits of the form $\dfrac{0}{0}$ or ∞^0 must yield the value of 1.
7. If f' is bounded, then f is bounded.
8. If the function f is defined by

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

then f is differentiable and f' is unbounded on $[-1, 1]$.

9. If $n = 0, 1, 2, \ldots$, then $n! = \displaystyle\int_0^1 (-\ln x)^n dx$
10. If the improper integral $\displaystyle\int_a^b f$ converges, then so does $\displaystyle\int_a^b f^2$.

Section II. Multiple Choice Questions (選擇題, 每題4分, 不需說明理由)

1. Determine whether the function $f(x) = \dfrac{\tan x}{x}$ is continuous at the given point $c = 0$. If the function is not continuous, determine whether the discontinuity is removable or nonremovable.
   (a) Discontinuous; removable, define $f(0) = 0$
   (b) Discontinuous; removable, define $f(0) = 1$
   (c) Continuous
   (d) Discontinuous; nonremovable

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2. Find y'' for $2y - x + xy = 7$.

   (a) $\frac{y+1}{(x+2)^2}$

   (b) $\frac{2y-2}{x+2}$

   (c) $\frac{2y-2}{(x+2)^2}$

   (d) $\frac{2y+2}{(x+2)^2}$

3. Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

   $R(x) = 3x$

   $C(x) = 0.001x^2 + 1.4x + 90$

   (a) 2200 units

   (b) 4400 units

   (c) 1600 units

   (d) 800 units

4. For a certain drug, the rate of reaction in appropriate units is given by $R'(t) = \frac{4}{t} + \frac{3}{t^2}$, where t is measured in hours after the drug is administered. Find the total reaction to the drug from $t = 1$ to $t = 6$.

   (a) 9.67

   (b) 3.67

   (c) 5

   (d) 1.84

5. $\int x8^x \, dx$

   (a) $x8^x - 8^x + C$

   (b) $x8^x \ln 8 - 8^x + C$

   (c) $8^x + \frac{x}{\ln 8} 8^x + C$

   (d) $\frac{x}{\ln 8} 8^x - \frac{1}{(\ln 8)^2} 8^x + C$

6. Find $\lim_{n \to \infty} \left(\frac{n-6}{n+6} \right)^n$

   (a) e^{12}

   (b) $e^{-1/12}$

   (c) e^{-6}

   (d) e^{-12}

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7. Which of the following statements is false?

    (a) The series $\sum a_n$ must have no negative terms in order for the Direct Comparison test to be applicable.

    (b) If $\{a_n\}$ and $\{b_n\}$ meet the conditions of the Limit Comparison test, then if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

    (c) The sequence $\{a_n\}$ and $\{b_n\}$ must be positive for all n to apply the Limit Comparison Test.

    (d) All of there are true.

8. Find the power series representation for

$$f(x) = \int_0^x e^{-t^2} dt$$

    (a) $x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \cdots$

    (b) $x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} - \cdots$

    (c) $x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} + \frac{x^9}{216} + \cdots$

    (d) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$

9. Derive a series for $\ln(1+x^2)$ for $x > 1$ by first finding the series for $\frac{x}{1+x^2}$ and then integrating.

    (a) $2\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{nx^{2n}}$

    (b) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{nx^{2n}}$

    (c) $\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n}}{nx^{2n}}$

    (d) $2\ln x + \sum_{n=1}^{\infty} \frac{(-1)^{n}}{nx^{2n}}$