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1. (16 %) Using the method of variation of parameters to solve the differential equation

$$y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1 \quad , \quad x > 0$$

2. (17 %) Using the method of Laplace Transformation to solve the initial value problem of $y(t)$

$$y'' - 6y' + 9y = t^2 e^{3t} \quad \text{with} \quad y(0) = 2 \quad , \quad y'(0) = 17$$

3. (17 %) For the matrix $A = \begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$,

(1) Find the eigenvalues of **A**. (5 %)

(2) Find an orthogonal matrix $\boldsymbol{P}$ such that $\boldsymbol{P^t AP}$ is diagonal. (7 %)

(3) Find the inverse of $\mathbf{A^2}$. (5 %)

4. (16 %) Show that a region T with boundary surface S has the volume $V = \frac{1}{3}\iint_S r\cos\phi \, dA$,

where r is the distance of a variable point P: (x, y, z) on S from the origin O and ϕ is the angle between the directed line OP and the outer normal of S at P. And then find the volume of a ball of radius a by means of the formula.

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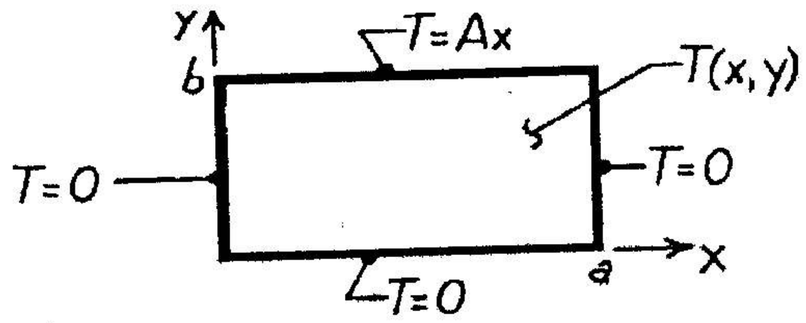
5. (17 %) A two-dimensional rectangular plate is subjected to the boundary conditions shown. Derive an expression for the steady-state temperature distribition T(x,y)



Steady state, 2D, constant properties conduction heat transfer problem.

Eq. => $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0,$

B.C. => T(0, y)= T(x,0)=T(a, y)=0, T(x,b)=Ax

T(x, y)=?

6. (17 %) We know that forced oscillations of a body of mass m on a spring are governed by the equation of

$$m\,y''+cy'+ky=r(t) \qquad \text{----(a)}$$

where k is the spring modulus and c is the damping constant. If the external force r(t) is a sine or cosine function and the damping constant is not zero, the steady-state solution represents a homonic oscillation having the frequency of external force.
Let m = 1gm, c= 0.02 gm/sec, and k= 25 gm/sec^2 , so that (a) equation become $y''+0.02y'+25y=r(t)$,
Where r(t) is measured in gm.cm/sec^2 . Let

$$r(t)=\begin{cases} t+\pi/2\text{ , when } -\pi<t<0, \\ -t+\pi/2\text{ , when } \quad 0<t<\pi \end{cases}$$

, $r(t+2\pi)=r(t)$,

Find the steady state solution y (t) ?
Hint: represent r(t) by a Fourier series, obtain the general solution of y (t)

(命題請用黑色鋼筆、原子筆繕寫或電腦打字；試題字體務求清晰，並一律以正面單頁書寫，背面請勿書寫。)