

1. Compressibility E of a fluid is defined as $E = \frac{-\Delta P}{(\Delta V)/V}$, the change in volume (V) of a fluid that is subjected to a change in pressure. Figure 1 shows a hydraulic linear actuator with inside diameter D and length L . A measure of **stiffness** of a linear actuator is the amount of the force required to cause a certain linear deflection. For two actuators that have inside diameters of $D=D_1$ and $D=D_2$ and lengths of $L=L_1$ and $L=L_2$ and that are filled with water.
- (a) Computer the stiffness ratio if $D_1/D_2=2$ and $L_1/L_2=2$. (10 %)
- (c) Describe how to design the stiffness of hydraulic linear actuator by varying diameter D and length L ? (10 %)

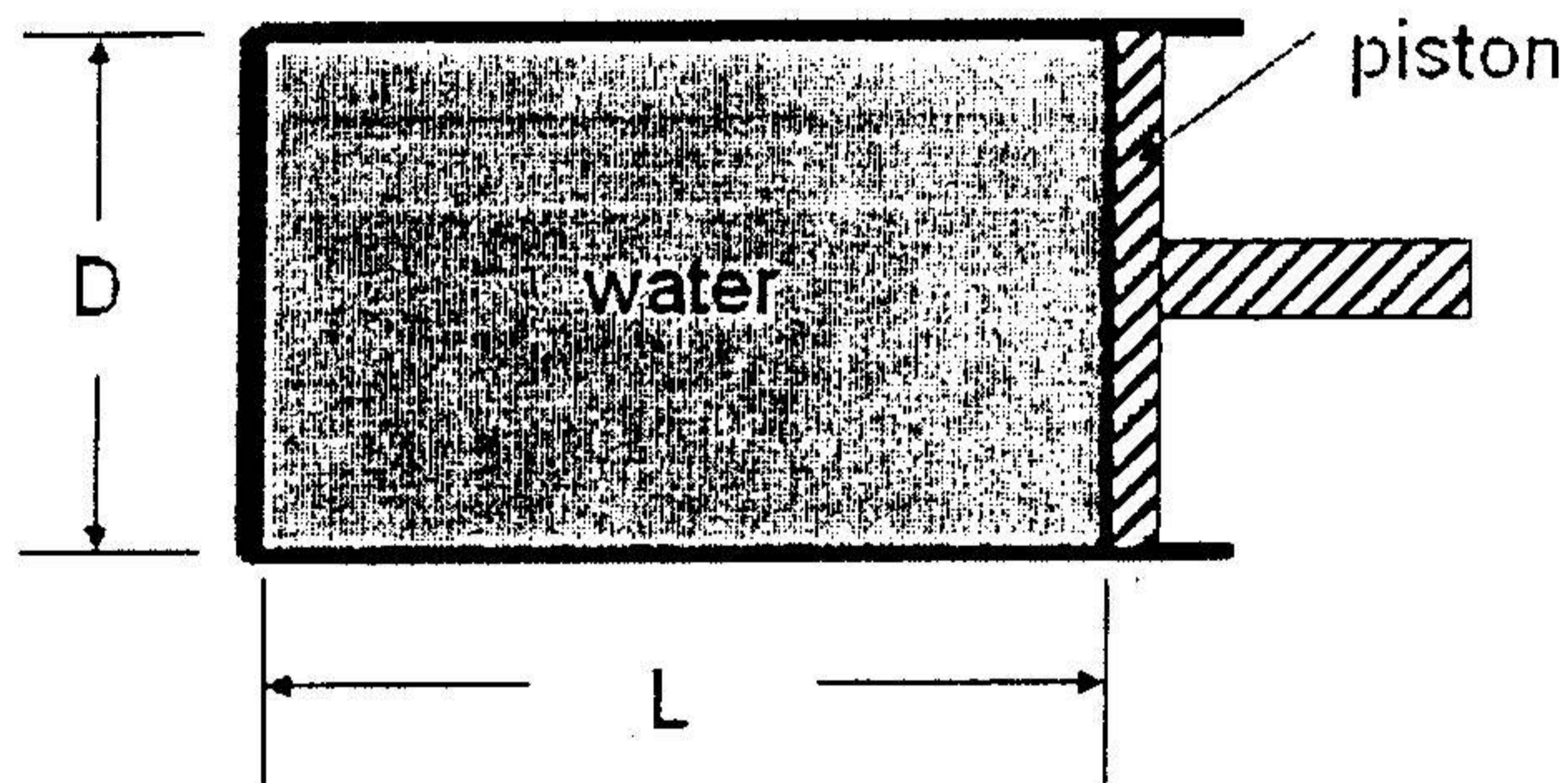


Figure 1

2. Figure 2 shows a tank (diameter D) with a smooth, well-rounded nozzle (diameter d) in the bottom through which fluid is discharging. Derive the equation for the time required to empty a tank, considering the variation of velocity as the depth decreases. (10%)

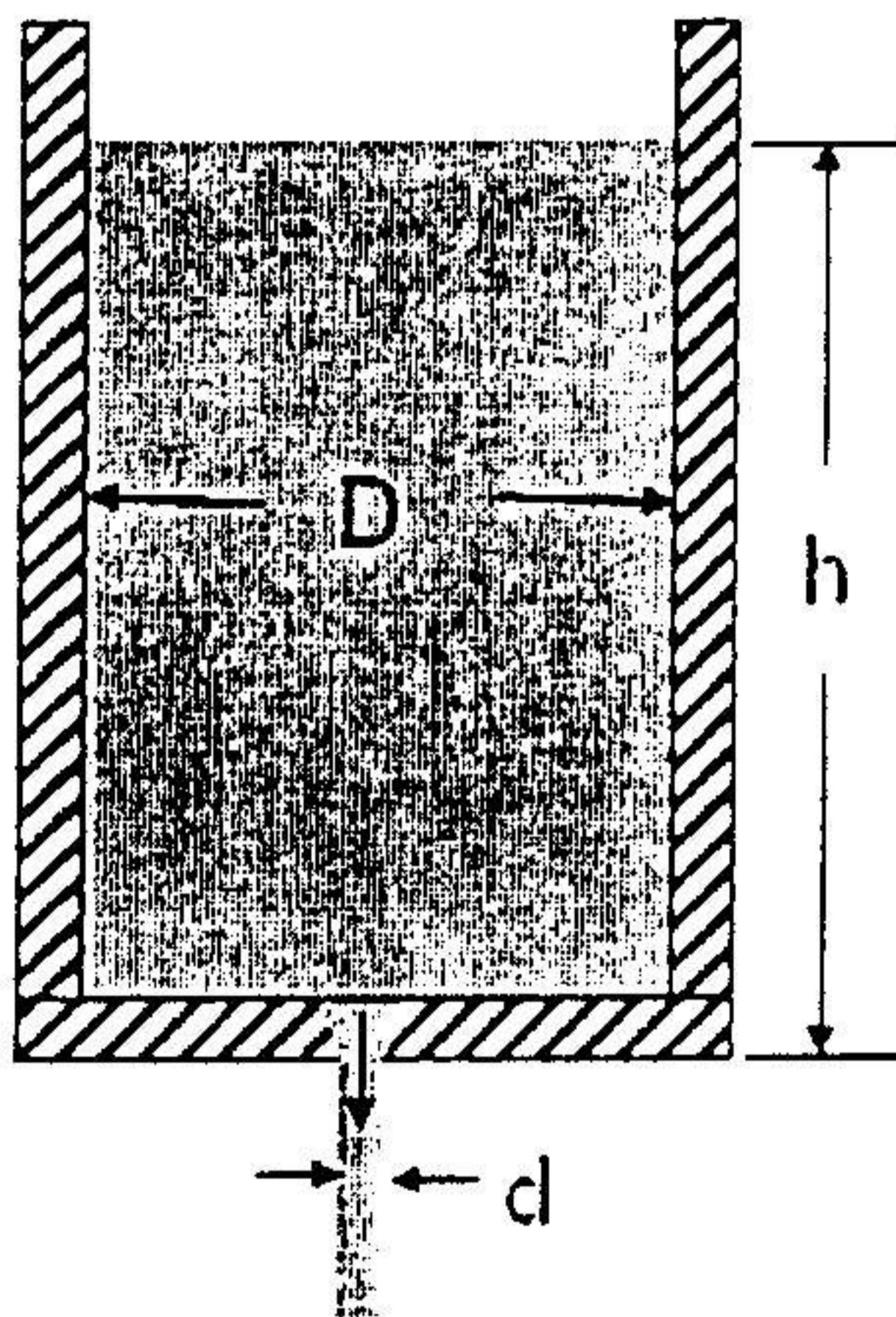


Figure 2

(背面仍有題目,請繼續作答)

3. Consider a two-dimensional flow field which is formed by a source of strength Λ located above a floor. The source is located at $(0, 1)$, as shown in Figure 3.

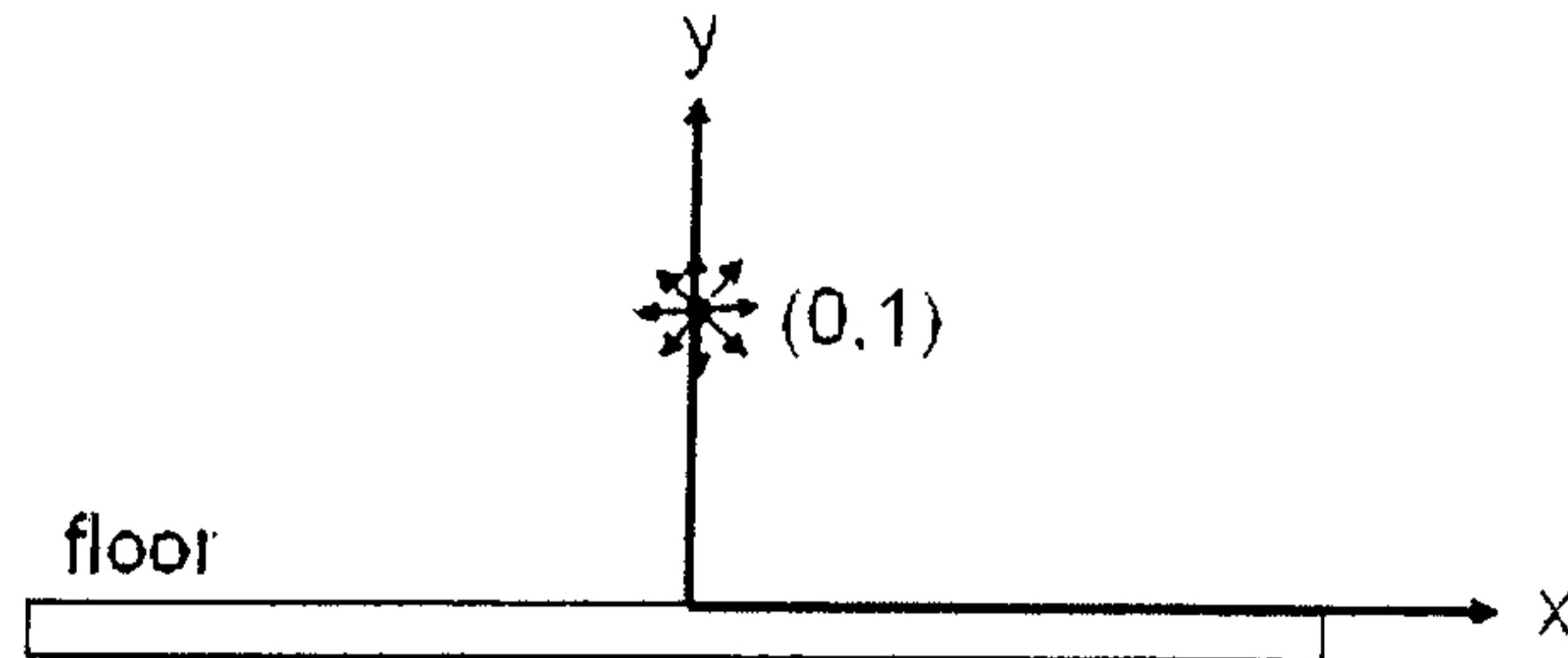


Figure 3

- (i) Determine velocity potential ϕ for this flow field. (10%)
- (ii) Locate the stagnation point in this flow field. (5%)
- (iii) Determine the streamline equation passing through the stagnation point in the Cartesian coordinate. (5%)
- (iv) Where does the maximum speed along the floor occur? (10%)
- (v) Assume the total (or stagnation) pressure in the flow field is P_0 and the density of the fluid is ρ , find the static pressure P at the point $(1,0)$. (10%)

Hint: If a source flow located at the origin $(0, 0)$, the velocities in (r, θ) direction of polar coordinate are

$$u_r = \frac{\Lambda}{2\pi r}, u_\theta = 0 \text{ where } r = \sqrt{x^2 + y^2}$$

4. As shown in Figure 4, the wind stress exerts a stress τ_w at the free surface which induces a Couette type of flow to the right. However, in order to conserve mass within the closed region, a return flow is required. In this problem, we can assume the flow is nearly parallel far from the ends and the free surface remains flat. Please answer the following questions:

- (a) Please simplify Navier Stokes equation and derive the governing equation for velocity flow field far from the ends. (10%)
- (b) List the boundary conditions and find out solution of the velocity flow field far from ends. (10%)
- (c) Find out the pressure gradient dp/dx in terms of τ_w and H . (10%)

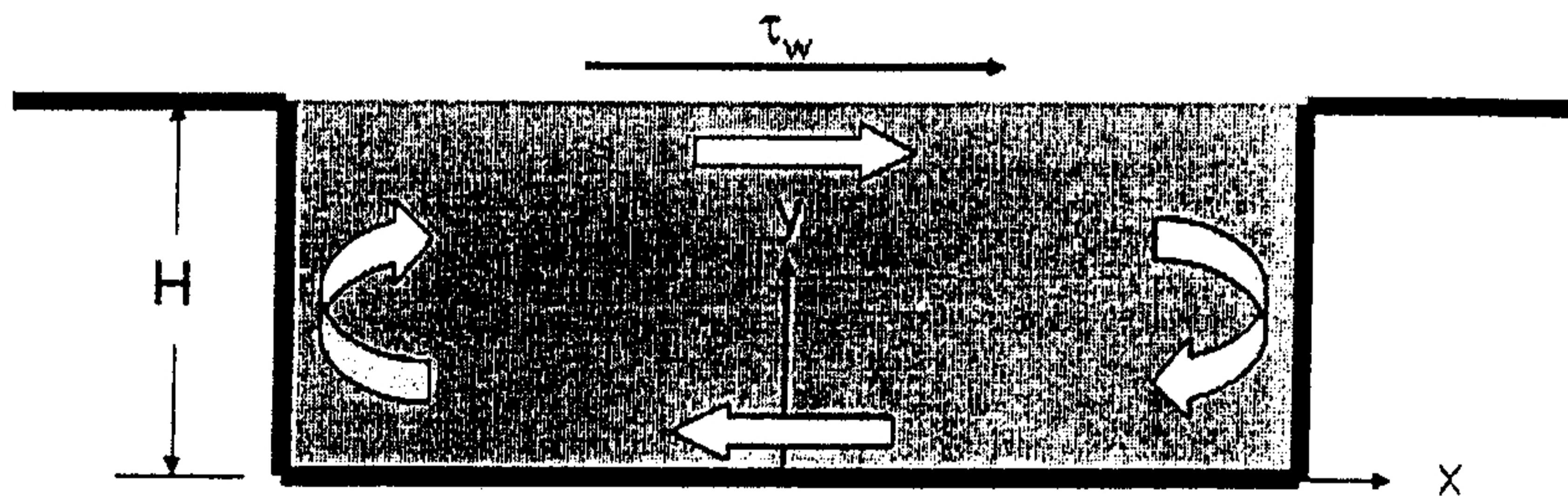


Figure 4