

國立清華大學 命題紙

97 學年度 統計學研究所 碩士班入學考試

科目 統計學 科目代碼 0103 共 3 頁第 1 頁 *請在【答案卷】內作答

選擇填充混合題：如果選項中有符合的答案，所有選項都必須選對才給分，如果沒有符合的選項則在答案卷上填入你自己的答案。（不須寫出計算過程。）

1. (5%) Let X_1, X_2, \dots, X_{12} be a random sample from a Poisson distribution with mean θ . Consider the test $H_0 : \theta = \frac{1}{2}$ vs. $H_1 : \theta < \frac{1}{2}$. We reject H_0 only when the observed value $Y = X_1 + X_2 + \dots + X_{12} \leq 2$. If $\gamma(\theta)$ is the power function of the test, what is the significance level of the test in terms of the power function?

(A) $1 - \gamma(\frac{1}{2})$ (B) $\gamma(12)$ (C) $\gamma(6)$ (D) your own answer _____

2. (5%) Let X_1, X_2, \dots, X_n be a random sample from a Weibull distribution and the probability density function for X_i can be written as $f(x, \theta) = 2\theta^{-2}x \exp(-\theta^{-2}x^2)$.

Please find a uniformly most powerful test for $H_0 : \theta \geq \theta_0$ vs. $H_1 : \theta < \theta_0$ with size α : _____

3. (5%) We are interested in fitting a simple linear regression for y versus x. The estimation is summarized as the following table.

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-1.3689	0.7959	-1.72	0.103
x	0.8049	0.8972	0.897	0.382
R-square	0.04			

Please choose the right description(s) from the following answers

- (A) The correlation between x and y is high since the coefficient of x is close to 1.
 (B) The degrees of freedom for the t-test is n-1 where n is the number of observations.
 (C) The width of the 95% confidence interval for the coefficient of x is greater than 1.
 (D) The regression sum of squares (SSR) must be close to zero since the R-square is close to zero.
 (E) None of the above answers

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97 學年度 統計學研究所 碩士班入學考試

科目 統計學 科目代碼 0103 共 3 頁第 2 頁 *請在【答案卷】內作答

4. (5%) A class of 400 students is divided into 2 sections of 200 each. Both sections are given a common exam. The following is observed

	Average	SD
Section 1	83	10
Section 2	42	10

Suppose all 400 scores are combined as one list. The SD of the new list will be

- (A) smaller than 10
 (B) larger than 10
 (C) equal to 10
 (D) can't tell without knowing the entire list
5. (5%) If X follows a Gamma distribution and its cumulative density function is $F(X)$. Let $Y=F(X)$. What is the distribution of Y ?
- (A) Normal distribution
 (B) Beta distribution
 (C) Binomial(1, $F(\infty)$)
 (D) Extreme value distribution
 (E) Your own answer _____
6. (5%) In an admission test, the committee decided to take nine students and six of them are female. If the proportion of each gender is equal in all the candidates, we are interested in testing whether there is any bias towards female students. What is the p-value for the exact test in this case?

7. (8%) Let X_1, X_2, \dots, X_n be a random sample from $N(\mu, 18)$. If we want to test $H_0 : \mu = 10$ vs. $H_1 : \mu = 12$ with $\alpha = \beta = 0.1$. Please find the minimal sample size needed for this end. _____ ($z_{0.05} = 1.645, z_{0.1} = 1.282$)

8. (8%) Let X, Y and Z have the joint pdf

$$\left(\frac{1}{2\pi}\right)^{2/3} \left[1 + xyz \exp\left(-\frac{x^2 + y^2 + z^2}{2}\right)\right] \exp\left(-\frac{x^2 + y^2 + z^2}{2}\right),$$

where $-\infty < x < \infty$, $-\infty < y < \infty$ and $-\infty < z < \infty$. Please choose the right description(s) from the following answers.

- (A) X, Y and Z are dependent
- (B) X, Y and Z are pairwise independent
- (C) The marginal distribution for each of X, Y and Z is normal.
- (D) None of the above answers

9. (8%) Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of a random sample of size 5 from the uniform distribution having pdf $f(x; \theta) = 1/\theta$, for $0 < x < \theta, 0 < \theta < \infty$

Please choose the right statement(s) from the following answers.

- (A) $2Y_3$ is unbiased for θ
- (B) Y_5 is a sufficient statistic for θ
- (C) $2Y_3$ is more efficient than $E(2Y_3 | Y_5)$
- (D) None of the above answers

10. $Y \sim \text{LOGN}(\mu, \sigma^2)$ if $\ln Y \sim N(\mu, \sigma^2)$. Assume that $Y_i \sim \text{LOGN}(\mu_i, \sigma_i^2), i = 1, \dots, n$, are independent.

(A) (8%) Find the distribution of Y_1/Y_2

(B) (10%) Find $E\left[\prod_{i=1}^n Y_i\right]$.

11. If X are independent random variables following geometric distribution and

$$f(x) = pq^x \text{ for } x = 1, 2, 3, \dots, 0 < p < 1, q = 1 - p$$

- (A) (8%) Please find the maximum likelihood estimator of $P[X > k]$ for arbitrary $k = 1, 2, \dots$
- (B) (10%) Please find a UMVUE for $1/p$

12. (10%) If y_1, y_2, \dots, y_n are random samples from $N(\theta, 1)$, and \bar{y} is the sample mean, find the conditional distribution of y_1 given \bar{y} .