國立臺北大學九十七學年度碩士班招生考試試題

系（所）別：通訊工程研究所　　　　組　　別：甲組（系統組）

科　　目：機率　　　　第 1 頁 共 1 頁

□可 ☑不可使用計算機

The following problems may be answered in Chinese or English. You need to give all details in order to receive any point.

1. True or false. No point if without explaining.
   (a) (4 points) $P[AB] = 1$ if $P[A] = P[B] = 1$.
   (b) (4 points) If the pdf $f_X(x)$ of a random variable X satisfies $f_X(x) = f_X(-x)$, the expectation $E[X]$ is equal to zero.
   (c) (4 points) If X and Y are random variables of zero mean, the covariance $\mathrm{COV}(X+Y, X-Y) = 0$.
   (d) (4 points) If X and Y are independent normal random variables with mean m and variance σ^2, $X+Y$ is independent of $X-Y$.
   (e) (4 points) If the joint pdf of random variables X and Y are given by $f_{XY}(x,y) = 2e^{-x-y}$ for $0 \le y \le x < \infty$, X and Y are independent.

2. (15 points) Suppose the random variable X is exponentially distributed with pdf $f_X(x) = e^{-x}$, $x > 0$. Find $P[X^3 - 6X^2 + 6X - 6 > 0]$.

3. (20 points) Suppose the random variable X has pdf

$$f_X(x) = \lambda^2 x e^{-\lambda x}, x > 0, \lambda > 0.$$

   (a) (10 points) Find the mean and variance of X.
   (b) (10 points) Determine the function $g(x)$ such that the random variable $Y = g(X)$ is uniformly distributed on [illegible].

4. (15 points) Suppose the [illegible] of X and Y is

$$f_{XY}(x,y) = \begin{cases} kxy(4-x-y), & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

   Determine k and compute $E[X|Y = y]$.

5. (15 points) Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed random variables with mean m_X and variance σ_X^2, and let N be an integer-valued random variable independent of the X_k's. The mean and variance of N are m_N and σ_N^2, respectively. Let

$$S = \sum_{k=1}^{N} kX_k.$$

   Determine the mean of S.

6. (15 points) One of two coins is selected at random and tosses n times. The coins are known to have probabilities of heads p_1 and p_2, respectively. Assume that $p_1 > p_2$.
   (a) (6 points) Find the probability that coin 1 is tossed given that k heads are observed.
   (b) (9 points) Find a threshold T such that when $k > T$ heads are observed, coin 1 is probable, and when $k \le T$ are observed, coin 2 is probable.

試題隨卷繳交