國立臺北大學九十七學年度碩士班招生考試試題

系（所）別：統計學系　　組　　別：

科　　目：機率論　　第 1 頁 共 8 頁

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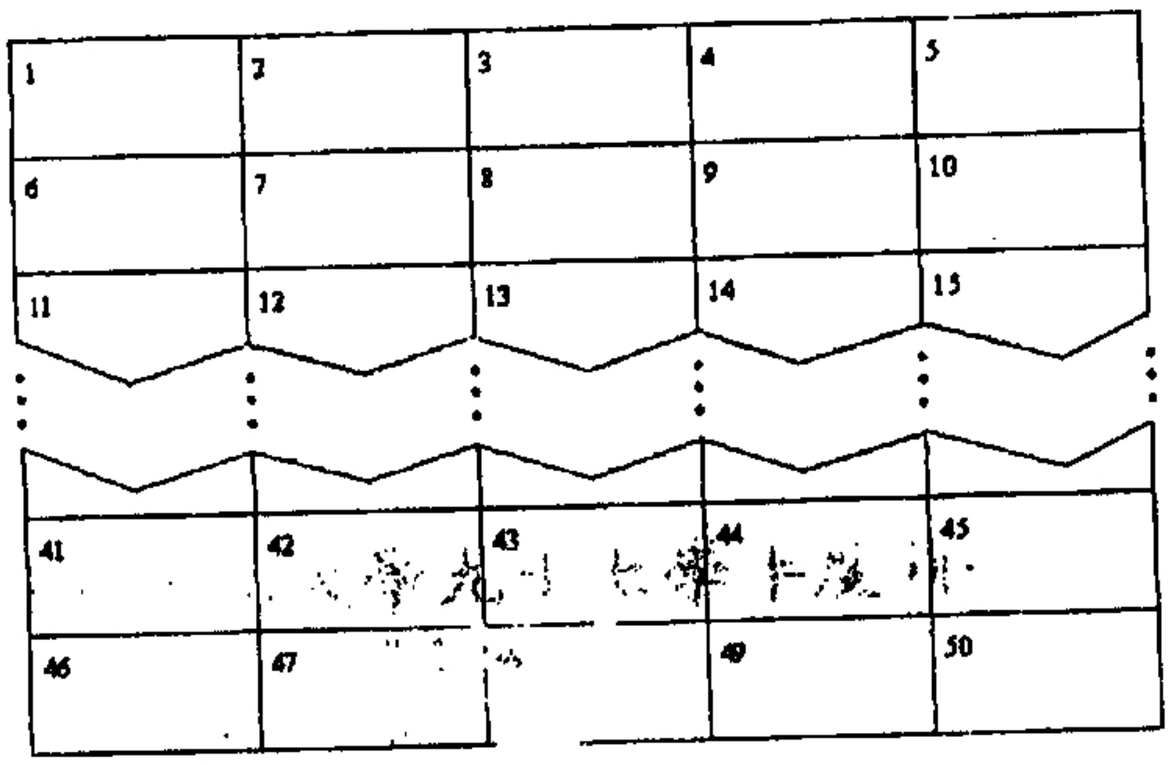
考試須知：

1) 含兩大部分（part I & part II），每一部分各 25 小題，合計 50 小題。每小題 2 分 （不倒扣）。

2) 群組題形，各小題定義等乃承 "該群組題意"。

3) 請於答案本上畫出如下表格（共 50 格），並請將答案填寫於正確位置方予計分。



4) 附表為常態分配。

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PART I

A. Suppose that A and B are two independent events for which $P(A)=0.4$, $P(B)=0.25$. Let A^C be the complement

1. $P(A|B)=$ (a) 0.4 (b) 0.25 (c) 0.15 (d) 0.35
2. $P(A\cap B)=$ (a) 0.1 (b) 0.15 (c) 0.25 (d) 0.35
3. $P(A\cup B)=$ (a) 0.9 (b) 0.65 (c) 0.55 (d) 0.35
4. $P(A^C\cap B)=$ (a) 0.6 (b) 0.25 (c) .5 (d) 0.45

B. Let n and k be two positive integrc : $n\geq k\geq 1$ and $\binom{n}{k}=\frac{n!}{k!(n-k)!}$.

5. $\binom{n-1}{k-1}=$ (a) $\frac{k(n-k)}{n}\binom{n}{k}$ (b) $\frac{k}{n}\binom{n}{k}$ (c) $\frac{n+1}{k+1}\binom{n}{k}$ (d) $\frac{k+1}{n+1}\binom{n+1}{k+1}$
6. $\sum_{k=0}^{n}\binom{n}{k}=$ (a) 0 (b) $2n$ (c) n^2 (d) 2^n
7. $\sum_{k=0}^{n}(-1)^k\binom{n}{k}=$ (a) 0 (b) $2n$ (c) n^2 (d) 2^n
8. $\sum_{k=0}^{n}k\binom{n}{k}=$ (a) 2^n (b) $n\cdot 2^{n-1}$ (c) $n\cdot 2^n$ (d) $(n+1)\cdot 2^n$

C. Let X be a Poisson random variable with parameter λ, $\lambda>0$. That is $P\{X=x\}=\frac{\lambda^x}{x!}e^{-\lambda}, x=0,1,2,\cdots\cdots$

9. If k is a positive integral, $\frac{P\{X=k\}}{P\{X=k-1\}}=$ (a) $\frac{\lambda}{k}$ (b) λ^k (c) $ke^{-\lambda}$ (d) $e^{-k\lambda}$
10. If $\lambda=\sqrt{30}$, what value of k maximizes $P\{X=k\}$? (a) 30 (b) 31 (c) 6 (d) 5
11. If $k=4$, what value of λ maximizes $P\{X=4\}$? (a) 2 (b) 4 (c) 6 (d) 16
12. If $\lambda=1$, the value of $P\{X>0\}$ is (a) e^{-2} (b) e^{-1} (c) $1-e^{-1}$ (d) 1
13. If $0<\lambda<1$, $E(X!)=$ (a) ∞ (b) λ (c) e^{-1} (d) $\frac{1}{1-\lambda}e^{-1}$

D. The probability density function of random variable X is

$$f(x)=c\cdot\exp[-\frac{(x-1)^2}{4}], \forall x\in(-\infty,\infty).$$

14. The value of c is (a) $\frac{1}{\sqrt{\pi}}$ (b) $\frac{1}{\sqrt{2\pi}}$ (c) $\frac{1}{2\sqrt{\pi}}$ (d) $\frac{\sqrt{2}}{\sqrt{\pi}}$
15. $Var[X]=$ (a) 1 (b) 2 (c) 3 (d) 4
16. $\int_{-\infty}^{\infty}f(x)dx=$ (a) $\frac{1}{\sqrt{\pi}}$ (b) $\frac{1}{\sqrt{2\pi}}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{\pi}}{2}$
17. $E(|X-1|)=$ (a) $\frac{1}{\sqrt{\pi}}$ (b) $\frac{2}{\sqrt{\pi}}$ (c) $\frac{1}{\sqrt{2\pi}}$ (d) $\frac{\sqrt{2}}{\sqrt{\pi}}$

E. Suppose X is a random variable with an exponential (λ) distribution, $\lambda>0$. The density function is $f(x)=\lambda e^{-\lambda x}, \lambda\geq 0$.

18. If $t>0$, $P\{X>t\}=$ (a) $e^{-\lambda t}$ (b) $\lambda e^{-\lambda t}$ (c) $1-e^{-\lambda t}$ (d) $1-\lambda e^{-\lambda t}$
19. If $s,t>0$, $P\{X>s+t\,|\,X>t\}=$ (a) $e^{-\lambda t}$ (b) $1-e^{-\lambda t}$ (c) $e^{-\lambda s}$ (d) $1-e^{-\lambda s}$
20. Apply $\Gamma(n)=\int_0^{\infty}t^{n-1}e^{-t}dt$, and $\Gamma(n+1)=n!$ to evaluate $E[X^n]$, $n=1,2,3,\cdots$. $E[X^n]=$

    (a) $\lambda(n!)$ (b) $\lambda^n(n!)$ (c) $\frac{n!}{\lambda}$ (d) $\frac{n!}{\lambda^n}$
21. The variance of X, $Var(X)=$ (a) $\frac{1}{\lambda}$ (b) $\frac{1}{\lambda^2}$ (c) $\frac{2}{\lambda^2}$ (d) $\frac{\sqrt{2}}{\lambda}$
22. If $\lambda=1$, the skewness of X is (a) 1 (b) 2 (c) 3 (d) 4

F. Let X and Y be random variables with joint density function $f(x,y)=\begin{cases}24xy, x\ge 0, y\ge 0, 0\le x+y\le 1\\ 0, \quad \text{otherwise}\end{cases}$

23. $P\{X<\frac{1}{2},Y<\frac{1}{2}\}=$ (a)$\frac{1}{4}$ (b)$\frac{3}{4}$ (c)$\frac{3}{8}$ (d)$\frac{1}{2}$

24. $P\{X>\frac{2}{3}\}=$ (a)$\frac{1}{9}$ (b)$\frac{2}{9}$ (c)$\frac{8}{27}$ (d)$\frac{10}{27}$

25. $P\{X<\frac{2}{3},Y<\frac{2}{3}\}=$ (a)$\frac{4}{9}$ (b)$\frac{7}{9}$ (c)$\frac{8}{9}$ (d)$\frac{7}{27}$

PART II

G. and Y are two random variables and F is denoted as the CDF of corresponding random variable(s).

26. If E(XY)=E(X)E(Y) and Var(X)=Var(Y), which of the following is TRUE?
   a) Corr (X,Y)=1.
   b) X and Y are independent.
   c) Cov (2X,3Y+2)=0.
   d) Fx,y(a,b)=Fx(a)Fy(b).

27. If Var(X)=1, Var(Y)=4 and Cor(X,Y)=.5, find Cov(2X+3Y+4, 3X+6)=?
   a) 9
   b) 6
   c) 15
   d) 21

28. If the jpdf (joint probability density function) f(x,y)=4xy, with the support 0<x<1, and 0<y<1, which of the following is TRUE?
   a) E(X)=1/2
   b) V(X)=1/9
   c) E(X|Y=y)=y/2
   d) V(X|Y=y)=1/18

H. If X1,…,Xn are independently and identically with uniform [0,1] distributed

29. Let Y=X1+…+Xn, which of the following is TRUE?
   a) Y is uniform (0,n) distributed.
   b) Y is approximately N(n/2, n/12) distributed as n large.
   c) Y is Gamma(2n,.5) distributed.
   d) None of the above.

30. Let $X_{(2)}$ be the second smallest (X1, …,X11) , which of the following is TRUE?
   a) $E(X_{(2)})>1/2$
   b) $V(X_{(2)})<1/12$
   c) $X_{(2)}$ is Gamma distributed.
   d) (a) (b) (c) are all true.

31. (Continue) Let M and N denote the median and mean of (X1, …,X11) respectively, which of the following is TRUE?
   a) E(M)<E(N)
   b) V(M)<V(N)
   c) (a) and (b) are both true.
   d) None of the above.

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I. X1 and X2 are two random variables with the bivariate normal joint density function

$$f(x_1,x_2)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\{-\frac{1}{2(1-\rho^2)}[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2+\left(\frac{x_2-\mu_2}{\sigma_2}\right)^2-2\rho\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2}]\}$$

32. Which of the following is FALSE?

a) The expected value of X_1 is μ_1.

b) The variance of X_1 is $(\mu_1^2-\mu_1)(\mu_2^2-\mu_2)$.

c) The correlation between X_1 and X_2 is ρ.

d) None of the above.

33. Which of the following is TRUE?

a) $E(X_1 \mid X_2=t)=\mu_1$.

b) $E(X_1X_2)=\mu_1\mu_2+2\rho\sigma_1\sigma_2$.

c) $Var(X_1 \mid X_2=t)=\sigma_1^2(1-\rho^2)$

d) $E(X_1^2 \mid X_2=t)=\sigma_1^2(1-\rho^2)+\mu_1^2$.

34. Which of the following is FALSE?

a) $f_{X_1|X_2}(x_1 \mid x_2)=f(x_1,x_2)/f_{X_2}(x_2)$.

b) $X_1 \mid X_2=t$ is normal distributed.

c) $f(x_2)=\int_{-\infty}^{\infty} f(x_1,x_2)dx_1$.

d) None of the above.

35. What is conditional probability density function of $X_1 \mid X_2=t$?

a) $\frac{1}{\sigma_1\sqrt{2\pi(1-\rho^2)}}\exp\{-\frac{1}{2(1-\rho^2)}[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2-2\rho\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2}]\}$

b) $\frac{1}{\sigma_1\sqrt{2\pi(1-\rho^2)}}\exp\{-\frac{1}{2(1-\rho^2)}\left(\frac{x_1-\mu_1-\rho(\sigma_1/\sigma_2)(t-\mu_2)}{\sigma_1}\right)^2\}$.

c) $\frac{1}{\sigma_1 2\pi(1-\rho^2)}\exp\{-\frac{1}{2\sqrt{(1-\rho^2)}}\left(\frac{x_1-\mu_1-\rho(\sigma_1/\sigma_2)(t-\mu_2)}{\sigma_1}\right)^2\}$

d) None of the above.

J. Let $M_X(t)$ denote the moment generating function of random variable X.

36. If $M_X(t) = 2^{10}(0.3e^t + .2)^{10}$, then which of the following is FALSE?

a) E(X)=6

b) V(X)=2.4

c) $E(X^2) = 38.4$

d) $P(X > 0) = (0.6)^{10}$

37. Which of the following is FALSE?

a) If Y=a+bX, then $M_Y(t) = e^{at} M_X(bt)$.

b) If X and Y are independent, then $M_{X+Y}(t) = M_X(t) + M_Y(t)$.

c) If $M_X(t) = e^{4(e^t-1)}$, then $P(X = 0) = e^{-4}$.

d) None of the above.

38. If $M_X(t) = e^{2(t+t^2)}$, Find the probability $P(|X - 1.5| < 1.5) = ?$

a) 0.19146

b) 0.34134

c) 0.53280

d) 0.68268

K. The number of people who enter the post office on a given day is a Poisson random variable with parameter λ. Among which, it is known that each person who enters the post office is a male with probability p and a female with probability $1-p$. Let X and Y denote, respectively, the number of males and females that enter the post office in one day.

39. What is the joint probability mass function of X and Y? $P(X = i, Y = j) = ?$

a) $P(X = i, Y = j) = e^{-\lambda} \frac{\lambda^{i+j}}{(i+j)!}$

b) $P(X = i, Y = j) = \binom{i+j}{i} p^i (1-p)^j$

c) $P(X = i, Y = j) = e^{-\lambda(1-p)} \frac{[\lambda(1-p)]^{i+j}}{(i+j)!}$

d) $P(X = i, Y = j) = e^{-\lambda} \frac{\lambda^{i+j}}{(i+j)!} \binom{i+j}{i} p^i (1-p)^j$

40. If $\lambda = 100$ and $p = 0.6$, what is the probability that the number people who enter the post office will equal to or larger than 120?

a) 0.47725

b) 0.95450

c) 0.02275

d) 0.04450

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L. Let X be the number of items produced in a factory A during a week is a random variable with mean 65 and variance 10, Y be the number of items produced in a fa[illegible] B during a week is a random variable with mean 65 and variance 15, and Cov(X, Y)= -5.

41. Compute an upper bound on the probability of $P(|X-Y|>15)$.

a) 0.0667
b) 0.0765
c) 0.1334
d) None of the above.

42. Compute an upper bound on the probability of $P(X-Y<-15)$

a) 0.0625
b) 0.0664
c) 0.1328
d) None of the above.

M. If $X_i, i=1,\cdots,n$ are iid $N(\mu,\sigma^2)$ distributed.

43. What is the distribution of $2n\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{\sigma^2}$?

a) $\chi^2(2n)$
b) $\chi^2(n-1)$
c) Gamma(n/2,1/4n)
d) None of the above.

44. Let $\overline{X}=(X_1+\cdots+X_n)/n$, what is the expected value of $S^2=\frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n}$?

a) $n\sigma^2$
b) $(n-1)\sigma^2$
c) $(n-1)\sigma^2/n$
d) $n\sigma^2/n-1$

45. Which of the following is FALSE?

a) Cov($\overline{X}$, S^2)=0.
b) The statistics $\overline{X}$ and S^2 are not independent.
c) The statistics $\overline{X}$ is normal distributed.
d) The statistics S^2 is approximately normal distributed as n large.

N. Others

46. Which of the following is FALSE?

a) E(E(X|Y))=E(X)

b) Var(X)=+Var[E(X|Y)]- E[Var(X|Y)]

c) $E[Var(X \mid Y)] = E(X^2) - E[E(X \mid Y)^2]$

d) $Var[E(X \mid Y)] = E[E(X \mid Y)^2] - [E(X)]^2$

47. If X and Y are independent standard normal random variables, which of the following is also independent?

a) X+Y and X-2Y

b) 2X+Y and X-2Y

c) X+Y and 2X-Y

d) X+Y and e^X

48. For random variables X and Y with correlation coefficient $\rho(X,Y)$. Which of the following is FALSE?

a) $-1 \le \rho(X,Y) \le 1$.

b) If $Var(\frac{X}{\sigma_x} - \frac{Y}{\sigma_y}) = 0$, then $\rho(X,Y) = 1$.

c) $\rho(X,Y) = 0$, if and only if, X and Y are independent.

d) With probability 1, $\rho(X,Y) = -1$, if and only if $Y = aX + b$ for some constant $a, b, a < 0$.

49. For random variables X and Y with correlation coefficient $\rho(X,Y)$. Which of the following is FALSE?

a) $Var(3X + 2Y + 3) \ge 0$.

b) $Var(\frac{X}{\sigma_x} + \frac{Y}{\sigma_y}) \ge 0$.

c) $Var(\frac{X}{\sigma_x} - \frac{Y}{\sigma_y}) \ge 0$.

d) $Cov(\frac{X-\mu_x}{\sigma_x} - \frac{Y-\mu_y}{\sigma_y}) < \frac{Cov(X,Y)}{\sigma_x \sigma_y}$.

50. Let X and Y are two independent random variables. Which of the following is FALSE?

a) If X-Y and X+Y are uncorrelated then $Var(X) = Var(Y)$.

b) If $Var(X) = Var(Y)$ then X-Y and X+Y are uncorrelated.

c) $Cov(X, XY) = Var(X)E(Y)$.

d) None of the above.

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Appendix:

Curve from 0 to X of normal distribution

X	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28521
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

試題隨卷繳交