國立臺北大學九十七學年度碩士班招生考試試題

系（所）別：統計學系

科　　目：基礎數學

☐不可 ☑可使用一般計算機(不含工程用計算機)

1. Let A be an n×n matrix. Please show that A is nonsingular if and only if the column vectors of A are linear independent. (10%)

2. Let $A=\begin{bmatrix}0 & -1 & -1\\ -1 & 0 & -1\\ -1 & -1 & 0\end{bmatrix}$ and define the linear transformation $L:R^3\to R^3$ by $L(x)=Ax$.

   (1) Please orthogonally diagonalize A. (12%)

   (2) Please calculate A^{100}. (4%)

   (3) What is the determinant of A^{-1}? (4%)

   (4) Is L one to one? Onto? Please explain why. (4%)

   (5) Give a basis of R^3 with respect to which the matrix representing L is the diagonalized matrix obtained in (1). (3%)

3. Please calculate the determinant of $\begin{vmatrix}1 & 3 & 1 & 2 & 5\\ 2 & 6 & 3 & 4 & 10\\ 1 & 6 & 2 & 2 & 5\\ -2 & -7 & 5 & -3 & 7\\ -3 & -9 & 7 & -6 & 10\end{vmatrix}$. (6%)

4. Let A be an n×n matrix and let x, y be vectors in R^n. Show that $Ax\bullet y=x\bullet A^Ty$. (7%)

5. Consider the function $f(x)=x\sin(1/x)$.

   a. Please draw the graph of $f(x)$. Show that the graph is concave downward to the right of $x=1/\pi$. (10%)

   b. Find the Taylor series for the function $\int_0^x f(1/t)dt$ about $x=0$, and compute the radius of convergence. (15%)

6. The impact of inflation on a fixed pension can be severe. If P represents the purchasing power (in dollars) of a \$80,000 pension, then the effect of a 6% inflation rate can be modeled by the differential equation $\frac{dP}{dt}=-0.06P,\ P(0)=80{,}000$, where t is in years. Find the purchasing power of the pension after 35 years. Round to the nearest dollar. (15%)

7. Use Newton's method to approximate the x-value of a point of intersection of $f(x)=3x^2$ and $g(x)=4\cos x$. Start with $x=1$ and continue until two successive approximations differ by less than 0.001. (10%)

試題隨卷繳交