

# 國立臺北大學九十六學年度碩士班招生考試試題

系(所)別：自然資源與環境管理研究所

組別：甲組

科目：微積分

第1頁共1頁

可 不可使用計算機

1. Let  $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ .

(a) (7%) Find  $f'(x)$ , if  $x \neq 0$ .

(b) (7%) Find  $f'(0)$ .

(c) (6%) Discuss whether  $f'(x)$  is continuous at 0 or not.

2. Let  $f(x) = \frac{1}{x}, x > 0$  and  $P = \{1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2\}$  be a partition of interval  $[1, 2]$ .

(a) (8%) Find the lower sum  $L(f, P)$  and upper sum  $U(f, P)$  for  $f$  base on the partition  $P$ .

(b) (4%) Show that  $\frac{533}{840} < \ln 2 < \frac{319}{420}$ .

(c) (8%) Applied Simpson's Rule on  $P$  to find an approximation of  $\ln 2$ .

3. Let  $f(x) = |x^3 + x^2 + 4|, x \in R$ .

(a) (14%) Find all relative extrema of  $f(x)$ .

(b) (6%) Evaluate  $\int_{-3}^2 |x^3 + x^2 + 4| dx$ .

4. Let  $R_t = \{(x, y) | x^2 + y^2 \leq t^2\}, t > 0$ .

(a) (7%) Evaluate  $\iint_{R_t} e^{-(x^2+y^2)} dA$ .

(b) (8%) Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$ .

(c) (5%) Evaluate  $\int_0^\infty e^{-x^2} dx$ .

5. Evaluate the following integrals.

(a) (5%)  $\int_{-1}^2 \frac{1}{x^2} dx$

(a) (7%)  $\int_0^\infty \frac{1}{e^x + e^{-x}} dx$

(b) (8%)  $\int_0^{\sqrt{\pi}} \int_{x^2}^{\pi} \sqrt{y} \cos y dy dx$