國立臺北大學九十七學年度碩士班招生考試試題

系（所）別：財政學系　　組　　別：

科　　目：微積分　　第 1 頁 共 1 頁

□可 ☑不可使用計算機

I. (30%) Consider a function $f(x) = \sin x$.

(1) (10%) Compute the Taylor series for $f(x)$ at $x = 0$. [The nth term of the Taylor series has to be presented.]

(2) (10%) Show that this Taylor series converges to $f(x)$ for every real value of x.

(3) (10%) For what value of x can we replace $f(x)$ by the Taylor series with 3 degree with an error of magnitude no greater than 6.48×10^{-4}.

II. (20%) Suppose that f and g are continuous and that

$$\int_1^2 f(x)dx = 4, \int_1^5 f(x)dx = -6, \int_1^5 g(x)dx = -8, \int_5^2 g(x)dx = 2.$$

Find the following.

(1) $\int_2^2 g(x)dx$　(2) $\int_5^2 f(x)dx$　(3) $\int_5^1 f(x) - 2g(x)dx$　(4) $\int_1^2 3f(x) + 2g(x)dx$.

III. (14%) 某公司的資產 A (單位: 百萬元) 隨著時間 t (單位: 年) 而增加。

假設其關係爲

$$A(t) = 5t^2 + 100 \qquad 0 \le t \le 5$$

(1)試問最後三年資產的平均成長率爲若干？ (5 分)

(2)在 $t = 2$ 時，資產的成長率爲若干？又其相對於 A 的成長百分比爲若干？ (9 分)

IV. (20%) The function whose defining equation is $y = f(x) = \dfrac{x^3}{x-2}$

(1) find the derivative $f'(x)$ and $f''(x)$.

(2) find its asymptotes.

(3) find its concavity and inflection points

(4) sketch the graph of the function.

V. (16%) Evaluate following limits :

(1) $\lim_{x \to 0^+} (1+2x)^{\frac{1}{x}}$

(2) $\lim_{x \to 2} \dfrac{\sqrt{2+x} - \sqrt{3x-2}}{\sqrt{4x+1} - \sqrt{5x-1}}$

(3) $\lim_{n \to \infty} \sum_{k=1}^{n} \ln(\sqrt[n]{1 + \dfrac{k}{n}})$

(4) $\lim_{n \to \infty} \sum_{k=1}^{3n} \dfrac{n}{n^2 + k^2}$

試題隨卷繳交