

# 國立臺北大學九十七學年度碩士班招生考試試題

系(所)別：都市計劃研究所  
科 目：微積分

組 別：乙組  
第 1 頁 共 1 頁  
可 不可使用計算機

I. (24%) Consider a function  $f(x) = e^{-x}$ .

(a) (8%) Compute the Taylor series for  $f(x)$  at  $x = 0$ . [The  $n$ th term of the Taylor series has to be presented.]

(b) (8%) Show that this Taylor series converges to  $f(x)$  for every real value of  $x$ .

(c) (8%) For what value of  $x$  can we replace  $f(x)$  by the Taylor series with 3 degree with an error of magnitude no greater than  $5.4 \times 10^{-3}$ .

II. (30%) Determine convergence or divergence of the series. Give reasons for your answers.

$$(a) \sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}} \quad (b) \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \quad (c) \sum_{n=1}^{\infty} \frac{1}{n \ln(n+1)} \quad (d) \sum_{n=1}^{\infty} \frac{\ln n}{n} \quad (e) \sum_{n=1}^{\infty} \frac{(-1)^n (n^2 + 1)}{2n^2 + n - 1} \quad (f) \sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$$

III. (20%) Suppose that  $f$  and  $g$  are continuous and that

$$\int_1^2 f(x) dx = -4, \int_1^5 f(x) dx = 6, \int_1^5 g(x) dx = 8, \int_5^2 g(x) dx = 2.$$

Find the following.

$$(a) \int_2^2 g(x) dx \quad (b) \int_5^2 f(x) dx \quad (c) \int_5^1 f(x) - 2g(x) dx \quad (d) \int_1^2 3f(x) + 2g(x) dx.$$

IV. (16%) Let  $f(x) = \begin{cases} \sin x / x, & 0 < x \leq \pi \\ 1, & x = 0 \end{cases}$ .

(a) (6%) Show that  $xf(x) = \sin x, 0 \leq x \leq \pi$ .

(b) (10%) Find the volume of the solid generated by revolving the region bounded by  $f(x)$ , the  $x$ -axis, and the line  $x=0$  about the  $y$ -axis.

V. (10%) Find the length of the curve  $y = \frac{1}{3}(x^2 + 2)^{3/2}$  from  $x=0$  to  $x=3$ .