國立台灣科技大學九十七學年度碩士班招生試題

系所組別：工業管理系碩士班甲組、乙組、丙組

科　　目：統計學

(Total 100 Points.) *There are 5 Problems in this exam. Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.*

1. (20 points)

Let X and Y have the joint probability density distribution

$$f(x,y)=\begin{cases} e^{-x}, & 0<y<x<\infty, \\ 0, & otherwise. \end{cases}$$

(a) Find the marginal distribution of X and Y. (8 points)

(b) Find $P(Y>5\mid X<10)$. (6 points)

(c) Let $U=X-Y$, find the probability density function of U. (6 points)

2. (20 points)

(a) Find the moment-generating function of the random variable X having a chi-squared distribution with v degrees of freedom. (10 points)

(b) Use the moment-generating function in part (a) to find the mean and variance of the chi-squared distribution with v degrees of freedom. (10 points)

3. (10 points)

It is claimed that a new diet will reduce a person's weight by 4.5 kilograms, on average, in a period of 10 days. The following data table recorded the weights of 10 men who followed this diet before and after a 10-day period.

Man	1	2	3	4	5	6	7	8	9	10
Weight Before	58.5	60.3	61.7	69.0	64.0	62.6	56.7	63.6	68.2	59.4
Weight After	60.0	54.9	58.1	62.1	58.5	59.9	54.4	60.2	62.3	58.7

Use the signed-rank test at the 0.05 level of significant to test the hypothesis that the diet reduces the median weight by 4.5 kilograms against the alternative hypothesis that the median difference in weight is less than 4.5 kilograms. (Note: The critical values can be found in Table 1.)

4. (25 points)

Let $x_1, x_2, \ldots, x_n$ be an observed random sample from $N(\mu,\sigma^2)$, and let $A=(n-1)s^2/\sigma_0^2$

A size α test of $H_0:\sigma^2\le\sigma_0^2$ versus $H_a:\sigma^2>\sigma_0^2$ is to reject H_0 if $A\ge\chi^2_{1-\alpha,n-1}$.

Please find the power function for this test.

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5. (25 points)

Consider the simple linear regression model $y = X\beta + \varepsilon = [1 \quad x]\begin{bmatrix}\beta_0 \\ \beta_1\end{bmatrix} + \varepsilon$ and assume that these data are available: $y_1 = 4, y_2 = 3, y_3 = 5, y_4 = 6$; $x_1 = 3, x_2 = 2, x_3 = 3, x_4 = 5$

Find $X^T X$, $(X^T X)^{-1}$, $X^T y$, $\hat{\beta}_0$, and $\hat{\beta}_1$ (each with 5 points)

Table 1 Critical Values for the Signed-Rank Test

n	One-Sided $\alpha = 0.01$ Two-Sided $\alpha = 0.02$	One-Sided $\alpha = 0.025$ Two-Sided $\alpha = 0.05$	One-Sided $\alpha = 0.05$ Two-Sided $\alpha = 0.1$
5			1
6		1	2
7	0	2	4
8	2	4	6
9	3	6	8
10	5	8	11
11	7	11	14
12	10	14	17
13	13	17	21
14	16	21	26
15	20	25	30
16	24	30	36
17	28	35	41
18	33	40	47
19	38	46	54
20	43	52	60
21	49	59	68
22	56	66	75
23	62	73	83
24	69	81	92
25	77	90	101
26	85	98	110
27	93	107	120
28	102	117	130
29	111	127	141
30	120	137	152