國立台灣科技大學九十七學年度碩士班招生試題

組別： 資訊工程系碩士班

目： 計算機數學

分 100 分。（1. 是非題務必於答案卷內依序作答，在試題內作答不予計分；倒扣分數至該大題總分零分止。）

1. (10%) Please determine the number of integer solutions for solving $x_1 + x_2 + x_3 + x_4 < 15$, $x_1, x_2, x_3, x_4 > 0$.

2. (10%) Please solve the recurrence relation $T_n = 4T_{n-1} - 4T_{n-2}$ with boundary condition $T_1 = 6$ and $T_2 = 20$.

3. (10%) By using Chinese Remainder Theorem, please determine the general solution for solving the following three linear congruent equations: $x \equiv 1 \bmod 2$, $x \equiv 2 \bmod 3$, and $x \equiv 8 \bmod 15$.

4. (10%) Please transfer the logical expression $(P \vee Q) \longrightarrow R$ into the Conjunction Normal Form.

5. (10%) Find the vector in the column space of matrix A which is closest to the vector b where

$$A = \begin{pmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{pmatrix} \text{ and } b = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}.$$

Find a basis of the orthogonal complement of the column space of A.

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6. (10%) A pair of fair dice is continuously rolled until either a 1 or a 6 appears at which point the experiment stops. What is the probability that the experiment was stopped *not* because of 1's appearance?

7. (10%) Suppose P balls are distributed at uniformly random into N boxes. Let X_i be a random variable defined by $X_i = 1$ if box i is empty and $X_i = 0$ otherwise. Compute $E(X_i)$, the mean of X_i. What is expectation of the number of empty boxes after these P balls were distributed.

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8. (10%) Let P_3 be the vector space of polynomials of degree at most 3. Define inner product $< f, g >= \int_{-1}^{1} f(x)g(x)dx$ and $||f||^2 =< f, f >$. Find the angle between $x - 5x^3$ and $2 + 8x^2$.

9. True or false? (Every incorrect answer (unanswered questions excluded) gets -1% until 0% on this problem.)

   (a) (2%) The matrices $A_{n\times n}, B_{n\times n}$ are called similar if $B = Q^T AQ$ for some $Q_{n\times n}$.

   (b) (2%) There exists a square matrix with no eigenvectors.

   (c) (2%) There is exactly one inner product on the vector space R^n.

   (d) (2%) If $(A_1|b_1)$ is obtained from $(A|b)$ by a finite number of elementary row operations, then the systems $A_1X = b_1$ and $AX = b$ are equivalent.

   (e) (2%) If $A^2 = O$ then $A = O$, where O is the zero matrix.

10. (10%) Let $A = \begin{pmatrix} -1.8 & 0 & -1.4 \\ -5.6 & 1 & -2.8 \\ 2.8 & 0 & 2.4 \end{pmatrix}$. Find $\lim_{m\to\infty} A^m$.